HYDRO DYNAMIC AND MAGNETO HYDRO DYNAMIC FLOWS: AN ANALYTICAL STUDY

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CERTIFICATE

This is to certify that the thesis entitled "Hydro Dynamic and Magnetohydro Dynamic Flows : An Analytical Study" by **Sri Sheelvrat** for the award of the degree of Doctor of Philosophy of **Bundelkhand University, Jhansi,** is a record of bonafide research work carried out by him under my supervision and guidance. He worked under clause 7 of Bundelkhand University ordinance.

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CHAPTER I

CHAPTER I

INTRODUCTION

(A) HISTORICAL SURVEY

1. Fluid Dynamics

Inspite of present day, emphasis on atomic and nuclear physics and tendency of the theoreticians to avoid concrete models, mechanics still remain the corner stone of physics. The mechanics of fluids is the study in which the fundamental principles of general mechanics are applied to liquids and gases. These principles are those of the conservation of matter, the conservation of energy and Newton's law of motion. By the use of these principles we are enabled not only to explain and bring into rotation observed phenomenon but also predict at least approximately, the behaviour of the fluids under a set of specified conditions.

Towards the end of the Ninteenth century the Science of fluid mechanics began to develop into two directions which had practically no points in common. On the one side, these were the science of theoritical hydrodynamics which was evolved from Euler's equations of motion for a frictionless, non-viscous fluid and which achieved a high degree of completeness. Since, the results of this so-called classical sciences of hydro-dynamics stood in glaring contradication to experimental results, it had little practical importance. For this reason, practical engineers developed their own highly empirical science hydraulics. The

science of hydraulics was based on a large number of experimental data and differed greatly in its methods and in its objects from the science of theoretical hydrodynamics.

At the beginning of the present century L. Prandtl (1904) distinguished himself by showing how to unify these two divergent branches of Fluid dynamics. He achieved high degree of correlation between theory and experiment and paved the way for the remarkably successful development of fluid machanics which has taken place over the past seventyfive years.

Fluid mechanics, one of the oldest branches of physics provides the foundation for the understanding of many other aspects of the applied sciences and engineering. It is a subject of independent interest, in almost all field of engineering as well as in astrophysics, biology, biomedicine, meterology, physical chemistry, plasma physics and geophysics. The development of aeronautical, chemical and mechanical engineering. During the past few decades, on the one hand there was exploration of the space and on the other hand the study of fluid mechancis was stimulated and as a result it has become one of the most basic subject in engineering sciences at present.

2. Magnetohydrodynamics (MHD)

When a conductor carrying an electric current moves in a magnetic field it experiences a force tending to move it at right angle to the electric field. Conversely, when a

conductor moves in a magnetic field a current is induced in the conductor in a direction mutually right angle to both the field and the direction of motion. These two statements first enunciated by Faraday, constitute the laws of electromagnetism. The first is principle of electric motor, the second that of dynamo. This is nothing in them to suggest that the conductor must be solid. In fact suggestions have been made that the motion of the sea may produce perturbutions in the earth's magnetic field. Further tidal waves sweeping up the estuary of a river will cut the terrestial lines of force and generate a current which can be detected in a cable connecting two electrodes placed in the river on opposite banks.

In the case when conductor is either a liquid or a gas, electromagnetic force will be generated which may be of the same order of magnitude as the hydrodynamical and initial forces. Thus the equation of motion will have to take these electromagnetic forces into account as well as the other forces. The science which treats these phenomena is called magnetohydrodynamics (MHD), other variants of nomenclature are hydromagnetics magneto-fluid dynamics, magneto-gas dynamics etc.

Most liquids and gases are poor conductor of electricity. As a consequence their motions can normally be treated by principles of fluid-dynamics. However, it is possible to make some gases very highly conducting by

ionizing them. For ionization to take effect, the gas must be very hot at temperatures upwards of 5000 K or so. Such ionized gases are called plasmas. The material within a star is a plasma of very high conductivity and it exists within a strong magnetic field. Consequently we expert MHD effects to be realised in stars. Further at the engineering level, experiments have been made for electric generation by passing the ionized gas between the poles of a strong electromagentic, so that an electric current would be generated at right angle to the magnetic field, and so the direction of flow of the plasma, the current being collected by two spaced electrodes at right to the direction of the current flow. At the present time MHD generators are not a practical possibility owing to the difficulties of producing suitable, efficient and stable plasma, and sufficiently retractory materials to withstand the high temperature of the plasma.

In summary MHD phenomenon result from the mutual effect of a magnetic field and a conducting fluid flowing across it. Thus an electromagnetic force is produced in a fluid flowing across a transverse magnetic field and the resulting current and magnetic field combine to produce a force that resists the fluid's motion. The current also generates its own magnetic field which distorts the original magnetic field. An opposing or pumping force on the fluid can be produced by applying an electric field perpendicularly to the magnetic field. Disturbance in either

the magnetic field or the fluid can propagate in both to produce MHD waves, as well as up stream and downstream-wake phenomenon. The science of magneto fluid dynamics is the detailed study of these phenomenon, which occur in nature and are produced in engineering divices.

3. Historical Survey

The subject of MHD had its origin in the speculations of certain scientists concerning the magnetism of colestial bodies.

In 1989 Bigelow guessed that there were magnetic bodies on the Sun. Hale and Babcocks later confirmed this. In 1918 Zarmov made the attractive suggestion that magnetic field of the Sun and other heavenly bodies may be due to dynamo action, whereby the conducting material of the star acted as the armature and rotator of a self-exciting dynamo.

In the inter-war period the astrophysicists, L. Cowling and Ferraro began to explore the formal theory of MHD and its implications,with other scientists, and engineers like Williams and Hartman performed simple experiments on the flow of the conducting liquids in the laboratory.

In 1942 Alfven [1] discovered the simplest example of the coupling between the mechanical and magnetic field force in a highly conducting motion in an external magnetic field. He showed that this interaction would produce a new

kind of wave. He called it MHD wave. These waves run along the lines of forces with a velocity which varies directly as the intensity of magnetic field and inversely as the square root of the density of the liquid. Hence, transverse waves can be propagated in an electrically conducting incompressible fluid pervaded by a magnetic field and energy can be transmitted on the large scale exchange of fluid elements as much as the case of a non-conducting incompressible fluid.

To observe Alfven's wave motion in a laboratory Lundquist [25] used mercury as liquid which he placed in a stainless steel cylinder and applied external vertical magnetic field of about 1000 gauss parallel to its axis. Due to dissipation true standing wave could not be excited. In 1952 [21] tried liquid sodium as medium. Chapman and Ferraro [5] in early thirties developed the theory of 'magnetic storms'.

Ever since the appearance of work of Alfven and the interest in the subject has been growing rapidly and consequently, a lot of papers on MHD have appeared in the sequence of time.

In recent years, the study of MHD has gained and increased attention of mathematicians and engineers in view of wide applications in Plasma Physics and Astrophysics. Hartman [13] was the first to solve onthe flow of conducting fluid cross a magnetic field. He studied one-dimensional

flow between two infinite stationary insulated paralle plates and found the exact solution in closed form. The same work was extended by Hartman and Lazarous [14] who performed experiments with mercury. They took rectangular and circular pipe with transverse magnetic field and it was followed by an increase in pressure gradient. Nigam and Singh [35] have discussed that heat transfer for Hartman flow. Stability of such flows have been discussed by Lock [23], Michael [27], Pavelock and Tersov [37] and Stuart [46], Shercliff [41] studied the steady MHD flow in pipes and obtained exact solution for the axial flow across in a rectangular pipe for high Hartman number, and velocity profiles and found to degenerate into a core of uniform flows surrounded by boundary layers on each wall. Shercliff [42] solved the problem of circular pipe with conducting walls. He also calculated velocity profiles, pressure gradient and induced potential difference. The work of Hartman and Shercliff was extended by Chang and Lundgren [4]. They took MHD ducts found in a straight-pipe of arbitrary cross-section and conducting wall in the presence of transverse magnetic field.

An investigation of MHD axial flow in an annular region with radial magnetic field was taken by Globe [6] who found exact solution in closed form. In the next paper Globe [7] analysed the effects of axial magnetic field on the transition from laminar to turbulent flow for a pipe using mercury. Kapur and Jain [17] discussed the

investigation of Globe to a great extent.

As regards the study of unsteady flow of a conducting viscous incompressible fluid inthe presence of magnetic field, Lin [22] indicated a class of exact solutions of MHD equations for laminar flow while a theoretical formulation and investigation of unsteady flow was made by lady Zhenskaya and Solomika [55], Lud ford [24] and Regirer [38], Regirer considered the problem of unsteady MHD flow between to parallel walls with an existing transverse magnetic field neglecting the incudction effects outside the flow region.

Gupta [9] studied the flow of an electrically conducting fluid part in an infinite porous flat plate inthe presence of uniform transverse magnetic field. Kakutani [16] re-examined the problem studied by Gupta [9]. But he found that : an asymptotic solution for velocity and magnetic field is possible only if there is suction at the plate and suction is strong enough to check the outward transfer of vorticity due to Alfven wave. In these investigation the 'Hall' effect were neglected. The Hall effects later on were discussed by Sato [39], Yamanishi [55] and Sherman and Sutton [44], when the flow is through straight channel. Katagiri [19] investigated the Hall effects on magnetic hydrodynamic boundary layer flow on a semi-infinite flat plate.

Gold [8] has obtained an exact solution of steady one-dimensional flow of an incompressible, viscous,

electrically, conducting fluid through a circular cylinder in the presence of an applied (transverse) uniform magnetic field. He obtained the solution for all values of Hartman number. The corresponding problem discussed by Shercliff [43] was approximated by Gupta [10]. However, his solution was valid for small values of Hartman number. He has obtained the velocity profile, an induced magnetic field magnetic field by using interaction procedure upto second order term in the Hartman number. Gupta and Singh [12] have extended the problem of Gold to the unsteady flow for equal values of Reynold and magnetic Reynold's number. Later on Pathak [36] solved the same problem for general values of Reynold and magnetic Reynold's number. Laminar unsteady flow in an annulus under radial magnetic has been discussed by Gupta and Mittal [11] whereas Reynold's and magnetic Reynold's numbers are equal. For the first time, the effects of wall's porosity on pressure distribution and velocity were considered by Berman [2] who has studied two dimensional steady flow in a rectangular channel with porous walls and obtained a solution valid for small values of suction or injection. Later, Sellars [40] and Yuan [54] extended the work of Berman for large suction and injection values respectively. Morduchow [32] discussed the laminar flow through a channel and circular tube with injection by using the method of average. Berman [3] showed that for suction through walls of a porous circular tube. There were two solutions for hydrodynamically developed velocity profiles

for a certain range of wall Reynold's number and, in other range there was no solution. Terrill and Thomas [48] more recently showed that there exists dual solution for both blowing and suction, except in the range were Berman [3] has found solution. Verma and Bansal [50] have obtained an extra solution of Navier-Stokes equation for the flow of the fluid between two parallel plates, one at rest and another in uniform motion with uniform porous walls of different permeability for small and large suction. Terrill [47] gave a first order perturbation solution for the steady flow in a porous annulus of different permeability for small cross-flow Reynold's number. Huang [15] amended the above Terrill's paper by using the method of quasi-linerisation. Nanda [34] obtained exact solution of the Navier-Stokes equation and energy equation, for the case of a steady state of flow of the fluid through the annulus with porous walls while the inner walls moving with a constant velocity parallel to axis when other is at rest. Kapur and Malik [18] and Sinha and Choudhary [45] have studied the steady state laminar flow of the fluid between the coaxial porous cylinders rotating with constant angular velocity. Verma [51] has studied the pulsating viscous flow superposed on steady laminar flow in an annulus with porous walls. Verma and Gaur [52] have studied the unsteady flow in a porous annulus when flow takes place under the influence of a time varying pressure gradient from rest. Muhuri [32] has considered MHD flow between two parallel porous plates when one was given an impulsing or uniformly accelerated motion

with suction at both the walls. Mathur (26) has considered the unsteady flow of viscous incompressible fluid between two porous plates under a transverse magnetic field, when the rate of injection of the fluid at the lower plate is equal to the rate of suction at the upper plate.

Highly special though the property of irrotationality may seem to be, it is given great practical importance by the consequence of Kelvin's circulation theorem that material elements of a uniform field set into motion from rest remain without rotation unless they move into a region where viscous forces are significant. Recently Mittal (28, 29, 39) has made an attempt to find or create the condition under which the MHD flow becomes or tends to become irrotational in channel and ducts of different cross-sections under the action of an applied external magnetic field.

(B) Hydrodynamical Flows in Ducts

(a) Definition

The mechanics of fluids is the study in which the fundamental principles of general mechanics are applied to liquids and gases. These principles are those of the conservation of matter, the conservation of energy and Newton's laws of motion. By the use of these principles we are enabled not only to explain and bring into rotation observed phenomenon but also to predict

at least approximately, the behaviour of fluids under a set of specified conditions.

For mathematical simplicity in describing flow in which the influence of viscosity is small a hypothetical fluid having zero viscosity may be postulated. This is known as an ideal fluid.

To make headway in the study of fluid dynamics we must have a simplified model of the real fluid. As a model we take a continuous fluid and attribute to it at each point and instant a density ρ, a pressure, p, a velocity $\bar{v}$, a viscosity μ and a temperature T.

If is constant we have the case of incompressible flow of a homogenous fluid and it is particularly applicable to the motion of a liquid, whose compressibility is negligible.

The viscosity of real fluid never vanishes. Even if the viscosity is small, as in case of air and water, its effects are prominant near a boundary. For from rigid boundaries, viscous forces can in many cases be neglected. In some problems the assumption, the case of non-viscous flow, gives a reasonably satisfactory description of the flow of a real fluid far from a boundary. In other problems, however, the discrepancy between the solution $\mu = 0$ and that for very small may be quite striking.

(b) Steady or Stationary Flow

In general the parameters such as velocity, pressure and density, which describe the behaviour of a fluid, are not constant in a particular set of circumstances. They may vary from one point to another point, or from one instant of time to another, or they may vary which both position and time.

Steady flow is defined as that in which the various parameters at any point do not change with time. Flow in which changes with time do occur is termed unsteady or non-steady. Steady flow is simpler to analyse than unsteady flow. In practice absolutely steady flow is the exception rather than the rule, but many problems may be studied effectively by assuming that the flow is steady, for even though minor fluctuations of velocity and other quantities with time do in fact occur, the average value of any quantity over a reasonable interval of time remains unchanged.

A particular flow may appear steady to one observer but unsteady to another. This is because all movements are relative, any motion of one body can be described only by reference to another body, often by a set of coordinate axes. Since the examination of steady flow is usually much simpler than that of unsteady flow, reference axes are chosen, where possbile, so that the flow with respect to them is steady.

In general, fluid flow is three-dimensitional in the sense that the flow parameters-velocity, pressure and so on vary in all the three coordinate directions. Considerable simplification in analysis may often be achieved, however, by neglecting the coordinate directions so that appreciable variation of the parameters occurs in only two directions, or even in only one.

So-called one dimensional flow is that in which all the flow parameters may be expressed as functions of time and one space coordinate only. The single space coordinate is usually the distance measure along the central line (not necessarily straight) of some conduit in which the fluid is flowing. For instance, the flow in a pipe is frequently considered one-dimensional. In reality flow is never truly one-dimensional because viscosity causes the velocity to decrease to zero at boundaries.

In two-dimensional flow the flow parameter are functions of time and two rectangular space coordinates (say x and y) only. There is no variation in the z-direction and therefore, the same stream line pattern could at any instant be found in all planes in the fluid perpendicular to the z-direction.

Axially symmetric flow is three dimensional flow

such that with a suitable choice of cylindrical polar coordinates R, θ, Z every physical variable is independent of the angle θ or with a suitable choice of sphereical polar coordinates r, θ, $\emptyset$ every physical variable is independent of the angle $\emptyset$. Axisymmetric flow, although not two-dimensional in the sense defined above, may be analysed more simply with the use of two cylindrical coordinates (x and y).

(c) Equation of Continuity

The equation of continuity is really a mathematical statement of the principle of conservation of mass. This gives us an important formula which represents the conservation of mass and it is written in the form

$$\bar{q} + \frac{D\bar{q}}{Dt} = 0 \qquad (1.1)$$

(d) Euler's Equation of Motion

Consider a small rectangular element of non-viscous fluids with sides of length x, y, z. The forces acting on it are the pressure on its several faces and possibility a body force $\bar{F}$ per unit mass due to gravitational or some other external forces. The acceleration produced is $(\frac{D\bar{q}}{Dt})$ where $\bar{q}$ is the velocity at some point in its interior. The Euler's equation of motion is

$$\bar{F} - \frac{1}{\rho}\nabla p = \frac{Dq}{Dt} \qquad (1.2)$$

which in its alternative form becomes

$$\bar{F} - \frac{1}{\rho}\nabla p = \frac{1}{2}\nabla (q^2) + (\omega x\ q)\ x\ \frac{\partial q}{\partial t} \qquad (1.3)$$

(e) **Bernoulli's Equation of Motion**

The result of intergration of equation (1.2) is

$$\frac{P}{\rho} + \frac{u^2}{2} + gz = \text{constant}$$

or, if we divide by g and put $\rho g = w$, the specific weight of the fluid,

$$\frac{P}{w} + \frac{u^2}{2g} + Z = \text{constant}$$

This equation is generally known as the equation of pressure in steady flow. It was discovered by Danial Bernculli and it is of fundamental importance for the entire theory of hydro-dynamics.

(f) **The Navier-Stokes Equations**

It is clear that when viscous forces are acting in a liquid in motion, Euler's equation of motion will no longer hold good and that it must be replaced by a modified equation in which the viscous forces are represented by additional terms and these additional terms are in facts :

$$\frac{1}{\gamma}\nabla(\nabla \cdot \bar{q}) + \bar{q}$$

where is the coefficient of kinematic viscosity. In place of Euler's equation we then have the Navier-Stokes equation

$$\frac{Dq}{Dt} = F - \frac{1}{\rho}\nabla P + \frac{\mu}{3\rho}\nabla(\nabla \cdot q) + \frac{\mu}{\rho}\nabla^2 q \qquad (1.4)$$

(g) Reynold's Number

If we consider the motion of an incompressible, constant viscosity fluid in a gravity field, then the ratio $\dfrac{\text{inertia forces}}{\text{viscous forces}}$ is called Reynold's number and is denoted by R. Reynold's number is important whenever viscous forces influence fluid motions.

(h) Some Solvable Problems in Viscous Flow

There is no general solution to the Nevier-Stokes equations. Never-the-less there are some special problems which can be solved. Few of them, related to incompressible fluids, are considered here.

(i) Steady Motion between Parallel Planes

The region $0 \leqslant z \leqslant h$ between the planes $z = 0$, $z = h$ is filled up with incompressible viscous fluid (Fig.1.1). The plane $z = 0$ is held at rest and the plane $z = h$ moves with constant velocity $V_{\underset{\sim}{j}}$. It is required to determine the nature of the flow when conditions are steady, assuming that there is no slip between the fluid and either boundary, neglecting body forces.

Let $p(x, y, z)$ be any point within the fluid. Then the velocity $\bar{q}$ at p will be of the form

$$\bar{q} = v\,(y,z)_{\underset{\sim}{j}} \tag{1.5}$$

The equation of continuity $\nabla . \bar{q} = 0$, gives

$$\frac{\partial v}{\partial y} = 0 \tag{1.6}$$

and from (1.5) and (1.6) we infer that

$$\bar{q} = v(z)_j \tag{1.7}$$

with nobody forces, the Navier-Stocks vector equation may be taken in the form

$$\frac{\partial \bar{q}}{\partial t} + (\bar{q}.\nabla)\nabla \bar{q} = -\frac{1}{\rho}\nabla P + \nabla^2 \bar{q} \tag{1.8}$$

since the flow is steady $\frac{\partial \bar{q}}{\partial t} = 0$. Also

$$(\bar{q}.\nabla)\nabla \bar{q} = (\frac{\partial v}{\partial y})\ v(z)_j = 0$$

and

$$\nabla^2 \bar{q} = v\ (z)^j$$

Hence (1.8) gives

$$0 = -(\frac{\partial p}{\partial x} i + \frac{\partial p}{\partial y} j + \frac{\partial p}{\partial z} k) + \mu\ v\ (z)_j$$

equating coefficients of the unit vectors,

$$\frac{\partial p}{\partial x} = 0 \tag{1.9}$$

$$\frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2} \tag{1.10}$$

$$\frac{\partial p}{\partial z} = 0 \tag{1.11}$$

equations (1.10) and (1.11) show that p(y). Hence, (1.10) becomes

$$\frac{dp(y)}{dy} = \mu \frac{d^2 v(z)}{dz^2} \tag{1.12}$$

The L.H.S. of (1.12) is a function of y only, the R.H.S. is a function of z only. Hence each is a constant. As the fluid is moving in positive y-direction, the pressure p(y) should decrease as y increases. Hence $\frac{dp}{dy}$ $<$ 0 and so we take

$$\frac{dp(y)}{dy} = \mu \frac{d^2 v(z)}{dz^2} \tag{1.12}$$

The L.H.S. of (1.12) is a function is y only, the H.R.S. is a function of z only. Hence each is a constant. As the fluid is moving in positive y-direction, the pressure p(y) should decrease as y increases. Hence $\frac{dp}{dy}$

0 and so we take

$$\frac{dp(y)}{dy} = \frac{d^2 v(z)}{az^2} = - p$$

where P $>$ 0, solving for v gives

$$V(Z) = A + Bz - (\frac{P}{2\mu}) z^2 \tag{1.13}$$

when z = 0, v = 0 and when z = h, v = V, hence, we find

$$v(z) = (\frac{V}{h} z + \frac{ph}{2\mu} z^2 \tag{1.14}$$

(1.14) shows that the velocity profile between the plates is parabolic.

(ii) <u>Steady Flow through Tube of Uniform Circular Cooss-Section (Poiseuille Flow)</u>

Fig. 1.2 illustrates the steady flow of an

inviscid incompressible fluid through a circular tube of radius. P is a pointin the fluid having cylindrical polar coordinates (R, θ, z), referred to the origin 0 on the axis of the tube which is taken as z-axis. We assume that there are nobody forces. Then continuity considerations applied to an annular shaped element of radius, $R, R + \delta R$ of the fluid indicate that the fluid velocity is of the form

$$\bar{q} = w\ (R)\ k \tag{1.15}$$

Let us take the Navier-Stokes vector equation in form

$$\frac{\partial \bar{p}}{\partial t} + (\bar{q} - \frac{1}{\rho} p - x \quad (v \times \bar{p}) \tag{1.16}$$

For steady flow we have $\frac{\partial \bar{q}}{\partial t} = 0$

The equation (1.16) becomes

$$0 = -\frac{1}{p} \left[\frac{\partial p}{\partial R} \cdot R + \frac{\partial p}{R \partial \theta} k \right] + \frac{\partial p}{\partial z} \frac{}{R} \frac{d(Rw')}{dR} k$$

equating coefficients of unit vectors give

$$\frac{\partial p}{\partial R} = 0 \tag{1.17}$$

$$\frac{\partial p}{\partial \theta} = 0 \tag{1.18}$$

$$\frac{\partial p}{\partial z} = \frac{\mu}{R} \quad \frac{d(Rw')}{dR} \tag{1.19}$$

(1.17), (1.18) show that $p = p\ (z)$ so that (1.10) becomes

$$\frac{dp(z)}{dz} = \frac{\mu}{R} \quad \frac{d}{dR} \quad [Rw'(R)] \tag{1.20}$$

The L.H.S. of (1.20) is a function of z only; the R.H.S. is a function of R only. Hence, each is constant. As flow

is supposed to occur in positive direction, we suppose $(\frac{dp}{dz}) < 0$. Take each side of (1.20) to be - P, where P is a positive constant. Then

$$\frac{d(Rw')}{dR} = \frac{-PR}{\mu}$$

Or

$$R \frac{dw}{dR} = A - \frac{PR^2}{2\mu}$$

Hence

$$\frac{dw}{dR} = \frac{A}{R} - \frac{1}{2} \frac{PR}{\mu}$$

and so

$$w(R) = B + A \log R - \frac{1}{4} \frac{PR^2}{\mu} \qquad (1.21)$$

and w is finite on R = 0. Thus we require A=0, Also on R = a, w = 0 since there is no slip. Then = $\frac{1}{4}$ (Pa^2/μ).

Hence

$$w(R) = \frac{1}{4} \left(\frac{P}{\mu}\right) (A^2 - R^2) \qquad (1.22)$$

The form (1.22) shows that the velocity profile is perholic, i.e., the plot of w against R from R=0 to a is of parabolic shape.

The volume of the liquid discharged over anysection per unit that is :

$$Q = \int_0^a w(R).\ 2\pi R.\ dR = \frac{\pi Pa^4}{B\mu} \qquad (1.23)$$

If p denotes the pressure difference at two points on the axis of the tube distance l apart, then p = p/l.

(iii) **Steady Flow between Concentric Rotating Cylinder**

Fig. 1.3 shows two concentric infinite cylinders of radii a,b ($b > a$) with viscous fluid in between. The inner cylinder is held at rest whilst the outer is rotated with constant angular velocity. Let (R, θ, z) be the cylindrical polar coordinates at point P in the fluid referred to some point O on the axis of symmetry. Then when the motion is steady the velocity at P will be of the form.

$$\bar{q} = RW(R)\ \theta \tag{1.24}$$

where $w(R)$ is the angular velocity of the liquid at radial distance R. The pressure at P will clearly be of the form

$$P = P\ (R) \tag{1.25}$$

This time we take the Navier-Stokes vector equation in the form

$$\frac{\partial \bar{q}}{\partial t} + \nabla(\frac{1}{2} q^2) - \bar{q} \times (\nabla \times \bar{q}) = \frac{p}{\rho} \tag{1.26}$$

for steady flow

$$\frac{\partial \bar{q}}{\partial t} = 0 \tag{1.27}$$

Equation (1.26) becomes

$$- Rw^2 R = - \frac{1}{} \frac{dp}{dR} R + (R \frac{d^2 w}{dR^2} + 3 \frac{dw}{dE})$$

Equating the coefficient of unit vector, gives

$$\frac{1}{\rho}\frac{dp}{dR} = Rw^2 \tag{1.28}$$

$$R\frac{d^2w}{dR^2} + 3\frac{dw}{dR} = 0 \tag{1.29}$$

From (1.29), we have, on multiplying by R^2

$$\frac{d}{dR}\left(R^3\frac{dw}{dR}\right) = 0$$

whence

$$\frac{dw}{dR} = \frac{A}{R^3}; \quad w = B - \frac{A}{2R^2}$$

Taking w = 0, when R = a, w = , when R = b, we find that

$$w(R) = b^2(1 - a^2R^{-2})(b^2 - a^2)^{-1} \tag{1.30}$$

The fluid velocity at radial distance R has magnitude

$$q = RW(R) = b^2(R^2 - a^2R^{-1})(b^2 - a^2)^{-1}$$

Thus the viscous stress at this location is

$$\mu\frac{dq}{dR} = \mu b^2(1 + a^2R^{-2})(b^2 - a^2)^{-1}$$

Hence, the couple per unit length on the inner cylinder due to viscous drag has moment.

$$(2\pi a \times) \times a \times (2\mu b^2(b^2-a^2)^{-1}) = 4\pi a^2b^2 , (b^2 - a^2)^{-1}$$

This result forms the basis of measuring μ for some liquid using a rotating visco-meter.

(iv) Steady Viscous Flow in Tubes of Uniform Cross-Section

Before leaving the subject of solvable viscous flow problems, we treat the case of incompressible

unaccelerated flow through a tube of any uniform cross-section. We neglect body forces. Thus with $(d\bar{q}/dt) = 0$, $\bar{F} = 0$, the Navier-Stokes vector equation becomes

$$\frac{1}{\rho} p - \nabla^2 \bar{p} = 0 \tag{1.31}$$

Let us work with fixes coordinates axes, Ox, Oy, Oz with Oz taken parallel to the flow so that $\bar{q} = w(x,y)k$.

Then (1.31) gives

$$p = \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) k \tag{1.32}$$

Equating the coefficient of the unit vectors,

$$\frac{\partial p}{\partial x} = 0 \tag{1.33}$$

$$\frac{\partial p}{\partial y} = 0 \tag{1.34}$$

$$\frac{\partial p}{\partial z} = \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \tag{1.35}$$

Equating (1.33) and (1.34) allow $p = p(z)$ so that (1.35) gives

$$\frac{dp(z)}{dz} = \mu\left[\frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2}\right] \tag{1.36}$$

The L.H.S. of (1.36) is a function of z only, the R.H.S. is a function of x, y only. Thus each of constant, say $-P$, the minus sign being taken as we expect P to decrease as a increases. Then the problem reduces to the solution of the partial differential equation

$$\frac{\partial^2 w(x,y)}{\partial x^2} + \frac{\partial^2 w(x,y)}{\partial y^2} = -p/\mu \qquad (1.37)$$

subject to w vanishing on the walls of the tube.

(i) <u>A uniquencess Theorem</u>

To obtain solution of equation (1.37) it is desirable to establish a uniqueness theorem. A form which is little more general than required here is following

If $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = f(x,y)$ at all points (x,y) of a region S in the plane Ox, Oy bounded by a closed curve and if f is prescribed at each point (x,y) of S and at each point then any solution w = (x,y) satisfying these conditions is unique.

Part C

MAGNETOHYDRODYNAMIC FLOWS

MHD is the science which deals with the motion of a highly conducting fluid in the presence of a magnetic field. The motion of the conducting fluid across the magnetic field and the action of the magnetic field on these current give rise to mechanical forces which modify the flow of the field. It is this coupling between the electro-magnetic and mechanical forces which characterises phenomenon.

(a) <u>Fundamental Equations in MHD</u>

A general discussion of MHD involves the formulation and examination of the following equations.

(i) the equations of the electromagnetic field (Maxwell's equations),

(ii) the mechanical equations embodying the effect of the electromagnetic forces as well as other forces on the motion,

(iii) the equation of continututy,

(iv) the equation of heat transport.

Also an equation of state and equations connecting the physical properties of the material with pressure and temperature must be considered.

Maxwell's equations [20] involves the following vectors and their rates of change; the magnetic induction vector $\bar{B}$, the displacement vector $\bar{D}$, the electric field intensity $\bar{E}$ and the density of electric content $\bar{J}$. Denoting di-electric constant by k, magnetic permeability by μ and using electromagnetic units throughout, we have

$$\bar{B} = \mu \bar{H}, \ \bar{D} = k\bar{E} \qquad (1.38)$$

Maxwell's equations are the induction equation

$$\frac{\partial \bar{H}}{\partial t} = - \text{curl } \bar{E} \qquad (1.39)$$

and Ampere's equation

$$\text{curl } \bar{H} = 4\pi\bar{J} + \frac{\partial \bar{D}}{\partial t} \qquad (1.39)$$

The following important subsidiary equations also hold

$$\text{div } \bar{B} = 0, \ \text{div } \bar{D} = 4\pi q \qquad (1.41)$$

where q denotes the electric charge density. The first of these equations signifies that there are no free magnetic poles. The second equation is Poisson's equation.

Differentiating the second of equation (1.38) with respect to time t, substituting for $\frac{\partial \bar{D}}{\partial t}$ from equation (1.40) and using the vector identity div curl $\bar{H} = 0$ we find

$$\text{div } \bar{J} = -\frac{\partial q}{\partial t} \tag{1.42}$$

This is the continuity equation for electric charge.

Let denote the density of the fluid, $\bar{v}$ the velocity of a fluid element, p the hydrostatic pressure and $\bar{F}$ the body force per unit mass. When the electromagnetic terms are included, the equations of motion of an electrically conducting liquid take the form

$$[\frac{\partial \bar{v}}{\partial t} + (\bar{v}. \ ) \bar{v}] = - \text{grad } p + \mu \bar{J} \times \bar{H} + \bar{F} \tag{1.43}$$

The force $\bar{F}$ includes viscosity terms and any external forces such as gravity.

In the case of a uniform incompressible fluid is constant. Then the equation of motion can be written as

$$\frac{\partial \bar{v}}{\partial t} = - \text{grad } (\frac{P}{} + \frac{1}{2} v^2) + \frac{\mu}{4\pi} (\text{curl } \bar{H}) \times \bar{H}$$

$$- (\text{curl } \bar{v}) \times \bar{v} + \bar{F} \tag{1.44}$$

Displacement currents have been neglected here, so that curl $\bar{H} = 4\pi \bar{J}$. This is useful from of the equation of motion.

The equation of continuity of the fluid, expressing the conservation of matter, can be expressed in either of the equivalent forms

$$\frac{\partial q}{\partial T} + \text{div} \; (\bar{v}) = \frac{\partial q}{\partial T} + \text{div} \; \bar{v} \qquad (1.45)$$

The equation of state and the heat equation. In the case of perfect gas, the equation of state can be written as

$$P = \frac{RT}{m} \qquad (1.46)$$

where T is the temperature, R is the universal gas constant and m is the molecular weight in terms of the hydrogen atom. For adiabatic changes the (p,)- relation is

$$\frac{d}{dt} (p/^{r}) = 0, \text{ i.e. } P/^{r} = \text{constant} \qquad (1.47)$$

where $r = c_p/c_v$ is the ratio of specific heats at constant pressure and constant volume respectively.

To these must be added the heat equation

$$T \frac{dS}{dt} = \frac{dW}{dt} \frac{P}{} \frac{d}{dt} = - \text{div grad } T + \frac{1}{} \Phi \qquad (1.48)$$

where S denotes the specific entropy, W the specific internal energy, the thermal conductivity, and Ø the total dissipative function.

(b) <u>The Magnetohydrodynamic Approximation</u>

Since the magnetohydrodynamic equations combine the full complexity of Maxwell's equations with the fluid dynamic equations, it is clear that they will be extremely difficult to solve in their general form.

Accordingly some simplifications must be introduced if we are to proceed at all. Such approximations are generally used in engineering MHD as also in questions relating astrophysics.

A number of approximations can be made which will be valid over a rather wide range of conditions.

The first is to neglect the displacement current in Maxwell's equations, in the event that unsteady motions are of interest.

Another useful simplification is to neglect the current flow due to the transport of excess charge as compared to the conduction current.

The third is that we may neglect the electrostatic body force in the equation of motion.

These three approximations can be shown to be valid, i.e. displacement currents can be neglected in Maxwell's equation; current flow due to charge transport is quite small and electrostatic body force can be neglected inthe equation of motion. All three taken together can be considered to be magnetohydrodynamic approximations.

(c) Forms of Basic Equations for Reference

For reference, the equations of motion are now written out in details. For incompressible fluid

(viscosity assumed quasi-constant), the equation in vector form is

$$\left[\frac{\partial \bar{v}}{\partial t} + (\bar{v}.\ )\bar{v}\right] = p + \mu_f\ {}^2\bar{v} - + {}_e\bar{E} + \bar{J} \times \bar{B}$$

which is modified from of equation (1.44). Here the force $\bar{F}$ (per unit mass) of (1.43) is replaced by ${}_e\bar{F}$ + $(\bar{J} x \bar{B})$. $\bar{B}$ μ, $\bar{H}$, $\bar{J}$ and $\bar{E}$ have their usual meanings as given above. Actually ${}_e\bar{E}$ represents the electric force present when where exists net space charge, ${}_e\bar{E}$, $\bar{J} X \bar{B}$ is the interaction force between the current and the magnetic field which becomes the predominant force in magnetofluid mechanics, is the gravitational potential, μ_f is viscosity as identified from μ , the magnetic permeabilitity.

<u>Cartesian Coordinates</u>

$$\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right)$$

$$= \frac{\partial p}{\partial x} + \mu_f\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

$$- \frac{\partial}{\partial x} + {}_eE_x + J_yB_z - J_zB_y$$

$$\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right)$$

$$= -\frac{\partial \bar{p}}{\partial y} + \mu_f\frac{\partial^2 v}{\partial x^2} + v\frac{\partial^2 y}{\partial y^2} + w\frac{\partial^2 v}{\partial z^2}$$

$$- \frac{\partial}{\partial y} + {}_eE_y + J_zB_x - J_xB_z$$

$$\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

$$= -\frac{\partial p}{\partial z} + \mu_f\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right.$$

$$- \frac{\partial}{\partial t} + {}_cE_z + J_xE_y - J_yB_x$$

Cylindrical Coordinates

$$(\frac{\partial v_r}{\partial r} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_r\frac{\partial v_r}{\partial z} - \frac{v^2_\theta}{r})$$

$$= - \frac{\partial p}{\partial r} + \mu_f(\frac{\partial^2 v_r}{\partial r^2}) + \frac{1}{r}\frac{\partial v_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2}$$

$$- \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial \theta_\theta}{\partial r}) - \frac{\partial}{\partial r} + {}_eE_r + J_\theta B_z - J_z B_\theta$$

$$(\frac{\partial v_\theta}{\partial r} + v_r\frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} - \frac{v_r v_\theta}{r})$$

$$= - \frac{1}{r}\frac{\partial p}{\partial \theta} + \mu_f(\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial v_\theta}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}$$

$$- \frac{v_\theta}{r^2} + \frac{\partial^2 v_\theta}{\partial z^2}) - \frac{}{r}\frac{\partial}{\partial \theta} + {}_eE_\theta + J_z B_r - J_r B_z$$

$$(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z})$$

$$= - \frac{\partial p}{\partial z} + \mu_f(\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r}\frac{\partial v_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial r^2}$$

$$- \frac{\partial}{\partial z} + {}_eE_z + J_r B_\theta - J_\theta B_r$$

Sperical Coordinates

$$(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v^2_\theta + v^2_\phi}{r}$$

$$= - \frac{\partial p}{\partial r} + \mu_f [\frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \frac{\partial v_r}{\partial r}) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial v_r}{\partial \theta})$$

$$+ \frac{1}{r^2 \sin \theta^2} \frac{\partial^2 v_r}{\partial \phi^2} - \frac{2 v_r}{r^2} - \frac{2}{r^2} \frac{\partial v_\theta}{\partial \theta} - 2 v_\theta \frac{\cos \theta}{r^2}$$

$$- \frac{2}{r^2 \sin \theta} \frac{\partial v_\phi}{\partial \phi}] - \frac{\partial}{\partial r} + eE_r + J_\theta B_\phi - J_\phi B_\theta$$

$$(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial t} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial e} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} + v_r v_\theta - \frac{v^2_\phi \cos \theta}{r})$$

$$= - \frac{1}{r} \frac{\partial p}{\partial r} + \mu_f [\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial v_\theta}{\partial r}) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial v_\theta}{\partial r})$$

$$+ \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v_\theta}{\partial \phi^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta}{r^2 \sin^2 \theta} - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial v_\phi}{\partial \phi}$$

$$- \frac{}{r} \frac{\partial \phi}{\partial \theta} + eE_\theta + J_\phi B_r - J_r B_\phi$$

$$(\frac{\partial v_\phi}{\partial t} + v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi V_r + v_\theta v_\phi \cot \theta}{r})$$

$$= \frac{1}{r \sin \theta} \frac{\partial p}{\partial \phi} + \mu_f [\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial v_\phi}{\partial r}) +$$

$$\frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial v_\phi}{\partial e}) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v_\phi}{\partial \phi^2} - \frac{v_\phi}{r^2 \sin^2 \theta}$$

$$+ \frac{2}{r^2 \sin^2 \theta} \frac{\partial v_r}{\partial \phi} + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial v_\theta}{\partial \phi}$$

$$- \frac{}{r \sin \theta} \frac{\partial}{\partial \phi} + e E_\phi + J_r B_\theta - J_\theta B_r$$

(d) Study of Magnetohydrodynamical Flows

(i) Two dimensional flow

Suppose of fluid moves in such a way that at any instant the flow pattern in a certain plane is the same as that in all other parallel planes within the fluid, the flow is said to be two-dimensional. If we take any one of the parallel planes to the plane z=0, then at any point in the fluid having Cartesian coordinates (x,y,z) all physical quantities (velocity, pressures, density, etc.)associated with the fluid are independent of z.

In the few cases of two-dimensional channel flow found in literature; the cross-sections considered are rectangular or circular. Although other cross-sections, that is, elliptical, parabolic, etc., might be solved in special cases, the rectangular and circular geometries have not been even austively studied.

(ii) One dimensional MHD Poiseuille flow

One of the simplest problems in magnetohydrodynamics, called the Hartmann problem, concerns the steady viscous laminar flow of an electrically conducting liquid between parallel planes in the presence of a magnetic field. The solution to this problem given us insight into the MHD generator,

pump, flowmeter, and bearings, and it forms the basic method for treating all viscous flow devices.

(iii) MHD Couette Flow

If, in a parallel-wall channel,one wall is stationary and the other is moving, we have that is generally referred to as couette flow. The difference between Poisseulle's and Couette flow arises by virtue of different boundary conditions for in Couette flow the fluid velocity at the top plate is non-zero where as it is zero in case of Poisseulle's flow. The top and bottom plates here are assumed to be insulators.

(iv) The rectangular channel

The general problem of MHD flow in a rectangular channel having walls of finite, non-zero conducting is a very difficult one, involving the magnetic field solutions not only in the conducting fluid but in space outside the channel as well. The problem of the steady flow of an in compressible, viscous, electrically conducting fluid through a rectangular pipe in the presence of a transverse applied magnetic field was solved by Shercliff [43] while a problem of a channel with ideally conducting walls was considered by Ulfyand [49] and later by Chang and Lundren[4].

(v) MHD flow in a circular cross-section pipes

In the study of MHD flow in tubes of rectangular

cross-section the applied magnetic field was normal to two of the walls and parallel to the other walls. The analogous problem for the circular piple, where the applied field is in the radial direction, has been treated by Pai [20]. Although it is possible to obtain a radial magnetic field between two concentric iron pipes, it is more interesting to discuss the problem of MHD flow through a circular pipes in a uniform magnetic field. This latter problem is two dimensional, where as the former is one-dimensitional. The latter problem is a practical interest because of the occurence of pipe flow through transverse magnetic fields in MHD pumps, generators and flow-meter.

(vi) **Flow between two coaxial rotating cylinders with a radial magnetic field.**

We now consider the steady flow of an incompressible conducting fluid between two concentric rotating cylinders composed of an insulating material. A Uniform radial magnetic field is applied throughout the flow region. The cylinders terminate at perfect electrodes which are connected through a load. The cylinders are long enough that axial variations are negligible, and since there is no preferred orientation for stable laminar flow, there can be no angular variations.

(e) **Development of Study of Flow Problems**

The historical aspects of the development of MHD

problems have already been referred to in part (a) of the present chapter. Here it will be more beneficial to sketch a brief account of the contributions of certain important researchers in order to take the problem more clear and approachable.

(I) **Laminar flow of a conducting viscous liquid between parallel walls with a transverse magnetic field [13]**

The steady rectilinear flow of an electrically conducting liquid along a uniform channel under the action of a uniform transverse magnetic field gives rise to some of the few linear problems of magnetohydrodynamics. Hartmann has considered a one-dimensional problem in which a liquid is constrained to flow horizontally between two infinite horizontal stationary plane walls. Taking rectangular axes OXYZ such that the boundary walls have equations $z = \pm L$, and assume that the fluid velocity is given by = $(v(z), 0,0)$ and the magnetic field by $B_o = (o,o,B_o)$ where B_o is constant (Fig.1.4), he found

$$v = \frac{v_o M \text{ Cosh } M - \text{Cosh } (M z/L)}{(M \cosh M - \text{Sinh } M)} \qquad (1.49)$$

where M is Hartmann number defined by

$$M = B_o L \left(\frac{\sigma}{\rho v} \right)^{1/2} \qquad (1.50)$$

The form of the Hartmann velocity profiles [13] for various values of M are shown in Fig. (1.5). The main effect of the transverse field is to generate electric

currents which retard the fluid in the central regions and accelerate the fluid near the boundaries thus flattening the parabolic profiles experiences in the absence of the magnetic filed. When $M \angle 1$, that is when the viscous forces are large compared with the electromagnetic forces, or the channel is very narrow, equation (1.49) tends to the limit

$$v = \frac{3v_o(L^2 - z^2)}{2L^2} \tag{1.51}$$

when $M >> 1$, that is for flows in wide channels or in the presence of strong magnetic fields, equation (1.51) tends to the limit

$$v = v_o \; [1\text{-exp} - M \; (L- |z| \;)/L] \tag{1.52}$$

Thus the velocity is approximately constant except in two boundary layers with thickness of order L/M. This result indicates that the velocity gradient near wall is of order M/L and thus it is to be expected that a large magnetic field may given rise to instability of the laminar motion near the walls. This, however, is not found to be the case in experiments; in fact that the presence of a sufficiently strong magnetic field appears to stabilize such a motion and tends to inhibit the on set of turbulent motions.

(ii) <u>Steady motion of conducting fluids in pipes under transverse magnetic fields</u> [31]

There are relatively few linear problems in

magnetohydrodynamics, the theory of the motion in the present of magnetic fields of fluids which conduct electrical currents according to Ohm's law. One such problem is that of steady, Incompressible, rectilinear flow alowng a uniform channel under a constant transverse magnetic field. It is a case of considerable practical interest because of the utility of induction flow-meters, which rely on the generation of a measurable potential difference in the fluid in a direction perpendicular to the motion and to the magnetic field. However, laminar flow occurs more rapidly under a magnetic field since turbulence tends to be damped by eddy currents.

Hartmann has solved the one-dimensional problem where the flow is between two parallel walls, the field being vartually infinite in directions perpendicular to the imposed transverse field. His solution is given by equation (1.49). The form of the velocity profiles, drawn by Sharcliff, for various values of M are shown in Fig.(1.6). The imposition of the transverse field generates current which related the fluid near the centre of the channel and acclerate the fluid near the walls, flattening the normally parabolic velocity profile until finally there is a core of uniform flow and an exponential boundary layer, whose thickness is of the order of L/M, at the walls. When M is large, boundary layers are found to occur on every wall.

(v) **Laminar flow of an electrically conducting incompressible fluid in a circular pipe [41]**

Now we consider the simple case of a laminar steady flow of an electrically conducting incompressible fluid in a non-conducting circular pipe under an external radial magnetic field which is independent of the axial position of the pipe. We assume that there is no excess electric charge ($_e$ = 0), that the pipe is infinitely long, and then both the velocity and magnetic field are axially symmetrical, i.e. independent of the angular position and without tangential components. Furthemore, we assume that velocity has only an axial component which depends entirely on the radial distance. We use the cylindrical coordinates r, Ø and z. Where z = 0 arbitarily closes. In the present problem we have the following relations :

$$q_r = q_\theta = 0, \; q_z = q_z(r)$$

$$H_\theta = 0, \; H_r = H_r(r), \; H_z = H_z \; (r,z),$$

$$p = p(r,z)$$

$$\frac{\partial}{\partial t}(\rho) = \frac{\partial}{\partial t}(\omega\mu) = 0 \qquad (1.53)$$

where subscripts r,Ø,z refer to components along r,Ø, z directions respectively, f (r) means functions of r only, and f(r,z) means fuctions of r and z both. ( ) means any quantity of the unknown.

After the velocity distribution $q_z(r)$ is known, Hz and P can be obtained immediately by simple quadraturs. One interesting point is that the variation of pressure in the z-direction is always linear no matter whether there is a magnetic field or not. Only with the external magnetic field. $H_o = 0$, the pressure varies with the radial distance, while in ordinary laminar Poiseulle flow, the pressure is constant for a given section. This point is also similar to that of turbulent flow.

The similar problem for two dimensional case has been solved by Lehnert. The general behaviour of the velocity distribution of these two problems are the same.

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CHAPTER II

CHAPTER-2

2.1 SOME SELF - SUPERPOSABLE FLOWS

In this Chapter some self superposable flows in Cylindrical ducts have been discussed. This paper has been accepted for publication in ACTA Ciencia Indica.

In literature, we find the flows mainly in ducts with circular and rectangular sections. Recently Mittal (5.6) and his Co-workers (7,8,9,10) have studied the flows in ducts with some non-costomary shape. In this chapter we have tried to find some flows in ducts of cylindrical shape. Phenomenon of superposability and self-superposability having been proved to be the most appalicable techniques in solving the hydrodynamic and magnetohydro-dynamic problems have been used here in bringing out very important conciusions about these flows.

2.2 ON SOME SELF-SUPERPOSABLE MOTIONS IN CYLINDRICAL DUCTS

ABSTRACT

In this paper an attempt has been made to find some self-superposable fluid motions of incompressible fluid in cylindrical ducts. No stress has been given on boundary conditions and the solutions thus determined contain a set of constants. Pressure distribution of some of such flows has also been discussed. The curves

along which the vorticity of the flow become constant have also been attempted. Some irrotational flows and the surfaces along which the flows show the tendency of irrotationality has also been attempted. The aim of this paper is to introduce a method of solving the basic equations of fluid dynamics in cylindrical system of coordinaes by using the property of sel-superposability.

INTRODUCTION

It was shown by Ballash (2) that when a flow with velocity q satisfies the condition

$$\text{curl } (\bar{q} \times \text{curl } \bar{q}) = 0 \text{ ----------(2.1).}$$

it becomes self-superposable. In the present paper the authors have attempted some fluid velocities for which ($\bar{q}$ X curl $\bar{q}$)= $\bar{p}$ (say) can be respresented as the gradient of a scalar quantity. Attention has been focussed on incompressible fluids only. For such fluids some velocities have been determined which satisfy the condition in cylindrical system of coordinates and for which $\bar{p}$ can be represented as the gradient of a scalar quantity ϕ (say). The solutions thus found must be of self superposable type. Each will have a set of constants which can be determined by the boundary conditions.

It has further been shown by Ballash (3) that if $\bar{q}_1$ and $\bar{q}_2$ are self superposable flows and $\bar{q}_1 \pm \bar{q}_2$ are also self superposable then $\bar{q}_1$ and $\bar{q}_2$ are mutually

superposable. By using this property some more self superposable flows have been determined. Attemps have also been made to find some curves along which the vorticity of the flow becomes constant and also the conditions of irrotationality.

In this paper a method will be introduced to solve the basic equations of fluid dynamics in cylindrical coordinates by using the property of self superposability. Mittal (5,6) Mittal <u>et</u> <u>al</u>. (7, 8, 9 & 10) have recently solved the questions for different coordinates systems.

FORMULATION OF PROBLEM

Let $\bar{q} \times \text{curl } \bar{q} = \bar{p}$ ....(2.2)

For incompressible fluids, we have

$\text{div } \bar{q} = 0$ ....(2.3)

Now if a solution of equation (2.3) be found in such a way that $\bar{p}$ can be represented as the gradient of a scalar quantity say , i.e..

$\bar{p} = \text{grad } \phi$ ....(2.4)

It will give a self-superposable flow. It has been shown by Agrawal (1) that such solutions will also satisfy the equations of motion for a steady flow. For determining a flow of liquid let us consider the flow in

cylindrical coordinates (r,θ,z). If q_r, q_θ, q_z be the components of $\overline{q}$ at any point (r,θ,z) in cylindrical coordinates (11), then in order to make equation (2.3) integrable we may consider the following cases :-

<u>CASE-I</u> Let $q_r = 0$, In this case equation (2.3) will be satisfied by a solution.

$$q = 0$$

$$q = A\ U\ (r) a\ W\ (z)$$

$$q = \frac{B\ U_1\ (r)\ V\ (\theta)}{r} \qquad \ldots\ldots\ldots(2.5)$$

Where $U\ (r)$, $U_1\,(r)$ are integrable fuctions of r. $V\ (\theta)$ and $W\ (z)$ the integrable functions of θ and Z respectively and A and B the constants.

For this fluid velocity, it can be shown that $\overline{p}$ can be represented by the gradient of a scalar quantity given by

$$\begin{aligned} \emptyset = \quad & A^2 W^2 \int U^2 .\, dr + A^2 W^2 \int UU_1\, dr \\ & - B^2 V^2 \int \frac{U_1^2}{r^3}\, dr + B^2 V^2 \int \frac{UU_1'}{r^2}\, dr \\ & + \frac{B^2 U_1^2}{r^2} \int VV'\, d\theta - ABUU_1W' \int vd\theta \\ & - \frac{ABUU_1V'}{r^2} \int Wdz + A^2U^2 \int WW'dz \ \ldots\ldots\ldots\ldots(2.6) \end{aligned}$$

here $U(r)$, $U_1\ (r)$, $V\ (\theta)$, $W(z)$ are represented by U, U_1, V and W respectively and U', U_1, V' and W' represent their differentials.

By choosing different suitable sets of values of U, U_1, V and W me may get a number of self-superposable fluid velocities. One of such veloctiy can be obtained by taking.

$$U_1 = U = r,\ V = \theta,\ W = Z,\ A = B \quad \ldots\ldots\ldots\ldots (2.7)$$

the fluid velocity will become

$$\left.\begin{aligned} q_r &= 0 \\ q_0 &= a\ r\ z \\ q_z &= \frac{A\ r\ \theta}{r} = A\theta \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.8)$$

and $\emptyset = A^2[\frac{1}{3}\ r^3 z^2 + z^2 r^2 + \frac{r^2}{2} + \frac{\theta^2}{2} - \frac{r^2\theta^2}{2}]$ ....(2.9).

If U, U_1, V, W are constants, then

$$\left.\begin{aligned} q_r &= 0 \\ q_0 &= C_1 \\ q_z &= \frac{D_1}{r} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.10)$$

$$\text{and } \emptyset = \frac{D_1^2}{2\,r^2} - \frac{C_1^2}{r} \quad \ldots\ldots\ldots\ldots\ldots (2.11)$$

<u>CASE-II</u>

(i) When $q_0 = 0$, In this case, the self superposable flows may be

$$q_r = \frac{A_1 V_1(\theta)\ W\ (z)}{r}$$

$$q_0 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ (2.12)$$

$$q_z = \frac{B_1 U_2(r) V_2(\theta)}{r}$$

and ϕ is given by

$$\begin{aligned}\phi = &-A_1 B_1 V_1 V_2 W_1' \int \frac{U_2}{r^2} dr - B_1^2 V_2^2 \int \frac{U_2 U_2'}{r^2} dr \\ &- B_1 V_2^2 \int \frac{U_2^2}{r^3} dr + \frac{B_1^2 U_2^2}{r^2} \int V_2 V_2' d\theta \\ &- \frac{A_1^2 W_1 W_1'}{r} \int V_1^2 d\theta + \frac{A_1 B_1 W_1 U_2'}{r} \int V_1 V_2 d\theta \\ &- \frac{A_1 B_1 W_1 U_2}{r^2} \int V_1 V_2 d\theta \\ &+ \frac{A_1^2 V_1^2}{r^2} \int W_1 W_1' dz \\ &- \frac{A_1 B_1 V_1 V_2 U_2'}{r^2} \int W_1' dz \\ &+ \frac{A_1 B_1 V_1 V_2 U_2}{r^3} \int W_1 dz \qquad \ldots\ldots\ldots\ldots\ldots\ldots (2.13)\end{aligned}$$

(ii)
$$\left.\begin{aligned} q_r &= \frac{A_1 \theta z}{r} \\ q_\theta &= 0 \\ q_z &= A_1 \theta \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots (2.14)$$

and

$$\phi = \frac{A_1^2 \theta^2}{2} + \frac{A_1^2 z\theta^3}{3} - \frac{A_1^2 \theta^2 z^2}{2r} + \frac{A_1^2 \theta^2 z^2}{2r^2} \qquad \ldots\ldots\ldots\ldots (2.15)$$

(iii) $$\left.\begin{aligned} r &= c_1/r \\ q_\theta &= 0 \\ q_z &= D_1 \end{aligned}\right\} \qquad \text{..................(2.16)}$$

and $$\phi = \text{const.} \qquad \text{..................(2.17)}$$

<u>Case-III</u> When $q_z = 0$, some self superposable flows may be

(i) $$\left.\begin{aligned} q_r &= \frac{A_2 V_3\ (\theta)\ W_2(Z)}{r} \\ q_\theta &= B_2\ U_3\ (r)\ W_3(Z) \\ q_z &= 0 \end{aligned}\right\} \qquad \text{..................(2.18)}$$

and $$\begin{aligned} \phi = {} & B_2^2\ W_3^2 \int U_3^2\ dr - B_2^2\ W_3^2 \int U_3\ U_3^1\ dr \\ & - A_2\ B_2\ W_2\ W_3\ V_3^1 \int \frac{U_3}{r^2}\ dr \\ & + \frac{A_2^2\ W_2\ W_2^1}{r} \int V_3^2\ d\theta \\ & - A_2 B_2 W_2 W_3 U_3^1 \int V_3\ d\theta \\ & + \frac{A_2^2\ V_3^2}{r^2} \int W_2\ W_2^1\ dz \\ & + B_2^2\ U_3^2 \int W_3\ W_3^1\ dz \qquad \text{..................(2.19)} \end{aligned}$$

(ii) $$\left.\begin{aligned} q_r &= \frac{A_2\ \theta\ z}{r} \\ q_0 &= \underline{A_2\ r_z} \\ q_z &= 0 \end{aligned}\right\} \qquad \text{..............(2.20)}$$

and $$\phi = 1/3\ A_2^2\ r^3 z^3 - A_2^2\ r^2 z^2 - A_2^2\ z^2 \log\ r$$

$$+ 1/3\, A_2^2 \; \theta^3 . z + \frac{A_2^2 \; \theta^2 \; z^2}{2\, r^2}$$

$$+ \frac{A_2^2}{2} \; r^2 \; z^2 \quad \ldots\ldots\ldots\ldots\ldots(2.21)$$

(iii)
$$\left.\begin{aligned} q_r &= C_2/r \\ q_\theta &= D_2 \\ q_z &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2.22)$$

and $\phi = D_2^2 \;\log r \quad \ldots\ldots\ldots\ldots\ldots\ldots(2.23)$

<u>Case - IV</u> When $q_r = q_\theta = 0$ a possible solution of equation (2.3) is given by

$$\left.\begin{aligned} q_r &= 0 \\ q_\theta &= 0 \\ q_z &= \frac{A_3 \; U_4 \; V_4}{r} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(2.24)$$

<u>Case - V</u> When $q_\theta = q_z = 0$, the self - superposable flow is given by

$$\left.\begin{aligned} q_r &= \frac{A_4 \; V_5 \; W_4}{r} \\ q_0 &= 0 \\ q_z &= 0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2.25)$$

<u>Case - VI</u> When $q_z = 0 = q_r$, the self - superposable flow is given by

$$\left.\begin{aligned} q_r &= 0 \\ q_0 &= A_4 \; U_5 \; W_5 \\ q_z &= 0 \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2.26)$$

In all the above cases U_n, V_n, W_n (n= 1,2,3,4---) are integrable functions of r, θ and z respectively and A_n, B_n, C_n & D_n (n = 1,2, ...5) are constants which may be determined by boundary conditions.

SUPERPOSABLE FLUID MOTION

It has already been shown that the hydrodynamic flows given by equations (2.5) and (2.25) are self superposable. It can also easily be shown that if $\bar{q}_1$ and $\bar{q}_2$ are the two flows given by equation (2.5) & (2.25) then $\bar{P}$ for $\bar{q}_1 \pm \bar{q}_2$ can be represented by the gradient of a scalar quantity. Thus $\bar{q}_1$ and $\bar{q}_2$ will be mutually superposable and a flow

$$\left.\begin{aligned} q_r &= \frac{A \quad V \quad (\theta) \quad W \quad (Z)}{r} \\ q_\theta &= \underline{B \; U \quad (r) \quad W \quad (Z)} \\ Q_z &= \frac{C \quad U \quad (r) \quad V \quad (\theta)}{r} \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots(2.27)$$

is possible. The same flow can be determined by mutually superposing the flows (2.12) & (2.26), (2.18) and (2.24).

PRESSURE DISTRIBUTION

It is interesting to note that ϕ is nothing but Bernoulli function given by (2.4).

$$\phi = (\frac{q^2}{2g}) + h = p/g \quad \text{.................(2.28)}$$

Where q, g, h and P denote velocity, acceleration due to gravity, height above some horizontal plane of reference and the pressure head.

It is well known fact that for an incompressible fluid the pressure head P is given by (2.4).

$$p = P/p_0 + \text{Constant}$$

Where P is the pressure distribution. Also if the motion of the fluid be steady and slow then the value of h can be taken without much loss of generality as r for the flows (2.5), (2.8) and (2.10), θ for the flows (2.12), (2.14),(2.16)and Z for (2.18), (2.20) and (2.22). Thus for

the flow (2.20), the pressure distribution is

$$P = K_1 Z + K_2 r^2 z^2 + K_3 \frac{\theta^2 z^2}{r} + K_4 \frac{\theta^2 z^2}{r^2} \quad \text{...(2.29)}$$

Similarly for the flow (2.22) taking $C_2 = D_2$ we have

$$P = K_8 \log r + K_9 (1/r^2+1) + K_{10} \quad \text{......(2.30)}$$

Where K_n ($n = 1,2,3,4,5,6,7,8,9$ & 10) are constants. Similarly the pressure distribution for the other flows can be determined.

VORTICITY OF THE FLOW

It was shown by Ballash [3] that for a self-superposable flow, vorticity is constant along its stream lines. It T is a unit tangent along a stream line then

$$\bar{T} \times q = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots(2.31)$$

by equation (2.16) and (2.31) it can be shown that

$$T = \left(0, \frac{r}{D_1\sqrt{1/C_1^2 + r^2/D_1^2}}, \frac{1}{C_1\sqrt{1/C_1^2 + r^2/D_1^2}} \right. \qquad \ldots(2.32)$$

Hence the vorticity of the flow (2.16) is constant along the curve represented by equation (2.32).

Similarly the curves of constant velocity can also be found for other flows.

IRROTATIONALITY

Vorticity for the flow (2.10) can be calculated as :-

$$\bar{\xi} = \frac{D_1}{r} \quad \hat{e}_2 + e_1\hat{k} \qquad \ldots\ldots\ldots\ldots\ldots(2.33)$$

It is clear from equation (2.33) that the flow (2.10) is not rotational.

For the flow (2.22).

$$\bar{\xi}_{\gamma} = 0$$

Hence the flow (2.22) is irrotational throughout.

Similar conclusions can be drawn for the other flows discussed earlier.

****

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CHAPTER III

CHAPTER - 3

MAGNETOHYDROSTATIC CONFIGURATION :

3.1 In this chapter a brief account of magnetohydrostatic problems has been given and force-free magnetic fields are defined. Definitions and scope of the subject have been given and explained. This chapter contain one research paper which is my own contribution. The paper is "On Some Magnetohydrostatic configuration in Cylindrical ducts.

3.2 MAGNETOHYDROSTATIC PROBLEMS

In general the action of the magnetic field on the electric currents flowing in a fluid will give rise to unbalanced mechanical forces which set up material motions. However, in certain cases the magnetic forces either vanish or may be balanced by fluid pressures. So that the conducting fluid is in equilibrium. When a fluid is at rest, so that $\bar{v} = 0$, the basic equations are

$$\operatorname{div} \bar{H} = 0 \qquad \ldots (3.1)$$

$$\frac{\partial \bar{H}}{\partial t} = \frac{1}{\mu_0} \quad \nabla^2 \bar{H} \qquad \ldots. (3.2)$$

$$\operatorname{grad} p = \rho \bar{F} + \frac{\mu}{4\pi} (\operatorname{curl} \bar{H}) \times \bar{H} \qquad \ldots (3.3)$$

Here $\bar{H}$ denotes the magnetic field, μ denotes the magnetic permeability, p denotes the pressure, ρ denotes the density and $\bar{F}$ denotes the body force per unit mass. The problem of magnetohydrostatics may be divided into two broad classes according as Lorentz forces does or

does not vanish, of particular interest is the case in which the magnetic field exerts no force on the material.

A VARIATION PROPERTY OF THE MAGNETIC FIELD

The lines of force of a magnetic field in an infinitely conducting fluid move with the field. It is clear that no matter how the field lines are moved about by the fluid motion, not all configurations can be distorted into each other. The problem of what properties make different hydromagnetic systems topologically equivalent is very difficult. Here we give a simple variational deviation of force-free magnetic fields, due to L, Woltzer [4].

Usually, the vector potential $\bar{A}$ of the magnetic field $\bar{H}$ is defined in such a way that div $\bar{A} = 0$, A is then unique. For present purpose. Let us ignore the unnecessary restriction on $\bar{A}$. Since

$$\bar{H} = \text{curl } \bar{A} \qquad \ldots(3.4)$$

and
$$\frac{\partial H}{\partial t} = \text{curl } (\bar{v} \times \bar{H}) \qquad \ldots (3.5)$$

we may write

$$\frac{\partial \bar{A}}{\partial t} = \bar{v} \times \text{curl } \bar{A} \qquad \ldots (3.6$$

For this particular form of the vector potential

$$\frac{\partial \bar{A}}{\partial t} = \text{curl } \bar{A} = 0 \qquad \ldots (3.7)$$

For a region ζ (enclosed by a stationary closed surface and writting,

$$I = \iiint_{\tau} \bar{A}.\ \mathrm{curl}\ \bar{A}\ d\tau,$$

we obtain

$$\frac{dI}{dt} = \iiint_{\tau} \left(\frac{\partial\bar{A}}{\partial t}.\ \mathrm{Curl}\ \bar{A} + \bar{A}.\mathrm{curl}\ \frac{\partial\bar{A}}{\partial t}\right) d\tau$$

Integrating by parts and using (3.7) we obtain

$$\frac{dI}{dt} = 2 \iiint_{\tau} \left(\frac{\partial\bar{A}}{\partial t}.\mathrm{Curl}\ \bar{A}\right) d\tau = 0 \qquad \ldots(3.8)$$

Integrating equation (3.8) we obtain

$$I = \iiint_{\tau} \bar{A}.\mathrm{curl}\ \bar{A}\ d\tau = \text{constant} \qquad \ldots(3.9)$$

For all vector potentials which are constant $\left(\frac{\partial\bar{A}}{\partial t} = 0\right)$ on the boundary. It follows that $\iiint_{\tau} \bar{A}$ and curl $\bar{A}\ d\tau$ is an invariant of the motion. Consider now the stationary values of the magnetic energy M, subject to the condition we have

$$M = \iiint \frac{H^2}{8\pi}\ d\tau$$

$$= \frac{1}{8\pi} \iiint_{\tau} (\mathrm{curl}\ \bar{A})^2 d\tau \qquad \ldots(3.10)$$

Let $\alpha/8\pi$ represent a Lagrangion Multiplier (the factor $1/8\pi$ is for convenience only). Then since I = constant, M is stationary if and only if $(M-\frac{\alpha}{8\pi})$ I is stationary.

Thus

$$\delta \iiint_{\tau} [(\mathrm{curl}\ \bar{A})^2 - \alpha\bar{A}.\mathrm{curl}\ \bar{A}]d\tau = 0$$

for all possible small changes $\partial\bar{A}$ in $\bar{A}$ such that $\partial\bar{A} = 0$ on the boundary. Thus

$$\iiint_{\tau} [2(\mathrm{curl}\ \delta\bar{A}).(\mathrm{curl}\ \bar{A})$$
$$-\alpha\delta\bar{A}\ \mathrm{curl}\bar{A} - \alpha\ \bar{A}.\mathrm{curl}\ \delta\ \bar{A}]\ d\tau = 0$$

Integrating by parts, since $\partial \bar{A} = 0$ on the boundary, we obtain

$$2 \iiint_{\tau} \partial\bar{A}.(\text{curl curl } \bar{A} - \alpha \text{curl} \bar{A}\; d\tau = 0$$

for arbitrary $\partial\bar{A}$ it follows that

$$\text{curl curl } \bar{A} = \alpha \text{curl } \bar{A}$$

or $$\text{curl } \bar{H} = \alpha \bar{H} \qquad \ldots(3.11)$$

The result holds for all magnetic field such that $\partial \bar{A} = 0$ or $\frac{\partial \bar{A}}{\partial t} = 0$ on the boundary. A magnetic field which satisfies equation (3.11) is known as a force-free magnetic field, since the Lorentz force $\frac{(\text{curl } \bar{H}) x \bar{H}}{4\pi}$ vanishes. Force-free fields are particular solutions of the magnetohydrostatic equation (3.3) in which the magnetic field does not disturb hydrostatic equilibrium. Equation (3.11) is the equation which governs beltrami fields in hydrodynamics.

3.3 MAGNETOHYDRODYNAMIC STUDY STATES OF AN INVISCID INFINITELY CONDUCTING FLUID

If $\bar{v}$ denotes the fluid velocity and ρ the density, the steady states consist of all solution of the equations.

$$\rho(\text{curl } \bar{v}) \times \bar{v} - \frac{\mu}{4\pi} (\text{curl } \bar{H}) \times \bar{H}$$

$$= -\text{grad } p - \rho \text{ grade } (\tfrac{1}{2}V^2) + \rho\bar{F} \qquad \ldots(3.12)$$

$$\text{curl } (\bar{v} \times \bar{H}) = 0 \qquad \ldots(3.13)$$

Here, the body force $\bar{F}$ will contain no frictional forces. The general solution of these equations is difficult and we shall content courselves by pointing

out that if ρ is constant, the configuration in which

$$\bar{v} = \pm \frac{\bar{H}\sqrt{\mu}}{\sqrt{4\pi\rho}} \qquad \text{....(3.14)}$$

is always a solution of these equations if $\bar{F} = 0$ (no external forces) and if $p + \frac{1}{2}\rho v^2 =$ constant. Note that these equations also reproduce the relation between $\bar{v}$ and $\bar{H}$ in hydromagnetic wave.

Chandrasekhar(2) and others have developed in full the axially symmetric form of equations (3.12) and (3.13) using stream functions for both the velocity $\bar{v}$ and the magnetic field $\bar{H}$.

3.4 ON SOME-MAGNETOHYDROSTATIC CONFIGURATION IN CYLINDERICAL DUCTS

ABSTRACT

In the present paper it is proposed to study some magnetic fields with conservative Lorentz force in cylindrical coordinates, neglecting the effects of displacement current. It has been shown that self-superposable flow of an electrically conducting incompressible fluid permeated by a magnetic field with conservative Lorentz force may constitute a magnetohydrostatic configuration under certain conditions. Some such possible configurations have been attempted for the conducting fluid flowering in cylindrical duct and pressure distributions of these configurations have also been determined.

INTRODUCTION

The flows of conducting fluids have been studied mainly in ducts which circular and rectangular sections when the walls of the ducts are non-conducting and the transverse magnetic field usually applied. In the present paper we shall deal with certain problems connected with the flow of conducting fluids in ducts having more general cross-sections and shapes for difference coordinate system. Phenomenon of superposability and self - superposability have been proved to be most applicable technique in solving hydrodynamic and magenetohydrodynamic problems. Here we have used these techniques frequently in some important conclusions about MHD duct flow. Mittal et al [7,8] have discussed some of such problems in ducts of different shapes and cross sections.

Lorentz force plays many important roles in MHD flows. Various authors [1,4,7] have represented it in various coordinate systems. It is also proposed to discuss some magnetic fields in cylindrical coordinates for which Lorentz force is conservative. Some magnetic fields whose Lorentz force can be represented by the gradient of a scalar quantity have been determined.

The basic assumption that displacement current field being small compared to the prevalent electric field can be discarded, have been taken into account in equations of the problem. The magnetohydrostatic

configurations can be developed when the magnetic fields with conservative Lorentz force will act upon self-superposable flow of electrically conducting fluids. It is attempted to find out the conditions of such magnetohydrostatic configurations. Some such type of magnetohydrostatic configurations have been studied by Mittal [7], Mittal, Thapaliyal and Agrawal [8], Mittal, Thapaliyal and Salam [9] Mittal and Khan [10] for the ducts of different shapes.

LORENTZ FORCE

Let the Lorentz force be represented by

$$\bar{L} = -\frac{\mu}{4\pi}\,(H \times \text{Curl } H) \qquad \text{.....(3.15)}$$

Here $\bar{H}$ and μ denote the magnetic field and permeability respectively. Now, suppose that ϕ be the required scalar so that we can express $\bar{L}$ as

$$\bar{L} = -\frac{\mu}{4\pi}\,\text{grad}\;\phi \qquad \text{.....(3.16)}$$

Writing the various terms of equation (3.15) & (3.16) in cylindrical coordinates (10), we get

$$L_r = \frac{H_\theta}{r}\left[\frac{\partial}{\partial r}(rH_\theta) - \frac{\partial}{\partial\theta}(H_r)\right] - H_z\left[\frac{\partial}{\partial z}(H_r) - \frac{\partial}{\partial r}(H_z)\right] \qquad \text{.....(3.17)}$$

$$L_\theta = \frac{H_z}{r}\left[\frac{\partial}{\partial\theta}(H_z) - \frac{\partial}{\partial z}(rH_\theta)\right] - H_r\left[\frac{\partial}{\partial z}(H_r) - \frac{\partial}{\partial r}(H_z)\right] \qquad \text{.....(3.18)}$$

$$L_z = H_r\left[\frac{\partial}{\partial z}(H_r) - \frac{\partial}{\partial r}(H_z)\right]$$

$$- \frac{H_\theta}{r}\left[\frac{\partial H_z}{\partial \theta} - \frac{\partial}{\partial z}(rH_\theta)\right] \quad ......(3.19)$$

Here (L_r, L_θ, L_z) and (H_r, H_θ, H_z) are components of $\bar{L}$ and $\bar{H}$ respectively at any point (r, θ, z) in cylindrical coordinates.

Also

$$\text{grad } \phi = \frac{\partial \phi}{\partial r}\bar{i}_1 + \frac{1}{r}\frac{\partial \phi}{\partial \theta}\bar{i}_2 + \frac{\partial \phi}{\partial z}\bar{i}_3 \quad .....(3.20)$$

Where $\bar{i}_1$, $\bar{i}_2$, $\bar{i}_3$ are unit vetors along r, θ & z directions respectively;

Equating the values of various tems from (3.15) & (3.16) we get

$$\frac{\partial \phi}{\partial r} = \frac{H_\theta}{r}\left[\frac{\partial}{\partial r}(rH_\theta) - \frac{\partial}{\partial \theta}(H_r)\right] - H_z\left[\frac{\partial}{\partial z}(H_r) - \frac{\partial}{\partial r}(H_z)\right] \quad ...(3.21)$$

$$\frac{\partial \phi}{\partial \theta} = H_z\left[\frac{\partial}{\partial \theta}(H_z) - \frac{\partial}{\partial z}(rH_\theta)\right] - rH_r\left[\frac{\partial}{\partial z}(H_r) - \frac{\partial}{\partial r}(H_\theta)\right] \quad ...(3.22)$$

$$\frac{\partial \phi}{\partial z} = H_r\left[\frac{\partial}{\partial z}(H_r) - \frac{\partial}{\partial r}(H_z)\right] - \frac{H_\theta}{r}\left[\frac{\partial H_z}{\partial \theta} - \frac{\partial}{\partial z}(rH_\theta)\right] \quad .....(3.23)$$

Values of $\frac{\partial \phi}{\partial r}, \frac{\partial \phi}{\partial \theta}, \frac{\partial \phi}{\partial z}$ when susbtituted

in $$\phi = \int \left(\frac{\partial \phi}{\partial r}dr + \frac{\partial \phi}{\partial \theta}.d\theta + \frac{\partial \phi}{\partial z}.dz\right) \quad .....(3.24)$$

will give ϕ.

Also the magnetic field $\bar{H}$ should satisfy the equation

$$\text{Div.}\; \bar{H}=0$$

or
$$\frac{1}{r}\left[\frac{\partial}{\partial r}(rH_r)+\frac{\partial}{\partial \theta}(H_\theta)+\frac{\partial}{\partial z}(rH_z)\right]=0$$

or
$$\frac{\partial}{\partial r}(rH_r)+\frac{\partial}{\partial \theta}(H_\theta)+\frac{\partial}{\partial z}(rH_z)=0 \qquad \ldots\ldots(3.25)$$

In order to make equation (3.24) integrable we may consider the following cases :

CASE-I When $H_r=0$, In this case equation (3.25) will be satisfied by the field.

$$\left.\begin{aligned} H_r &= 0 \\ H_\theta &= \frac{\alpha F(r) Z(z)}{r} \\ H_z &= \frac{\beta F_1(r)\Psi(\theta)}{r} \end{aligned}\right\} \qquad \ldots\ldots(3.26)$$

When $F(r)$, $F_1(r)$ are integrable functions of r, $\Psi(\theta)$ and $Z(z)$ the integrable functions of θ and z respectively and α and β the constant. For this magnetic field, it can easily be shown that Lorentz Force $\bar{L}$ can be represented by the gradient of a scalar quantity ϕ given by

$$\phi = \alpha^2\{Z(z)\}^2 \int \{F_1(r)\}^2 dr + \alpha^2 \{Z(z)\}^2 \int F(r)\, F'(r)\, dr$$
$$- \beta^2\{\Psi(\theta)\}^2 \int \frac{\{F_1(r)\}^2}{r^3} dr + \beta^2 \{\Psi(\theta)\}^2 \int \frac{F_1(r)\, F_1'(r)}{r^2} dr$$
$$+ \beta^2 \frac{\{F_1(r)\}^2}{r^2} \int \Psi(\theta)\, \Psi'(\theta)\, d\theta - \alpha\beta F(r) F_1(r)\, Z'(z) \int \Psi(\theta)\, d\theta$$
$$- \frac{\alpha\beta\, F(r) F_1(r) \Psi'(\theta)}{r^2} \int Z(z)\, dz$$
$$+ \alpha^2 \{F(r)\}^2 \int Z(z)\, Z'(z)\, dz \qquad \text{.....(3.27)}$$

Now it is clear that magnetic field (3.26) must have Lorentz conservative force as it is represented by the gradient of a scalar quantity ϕ given by equation (3.27)

By choosing different suitable sets of values of F (r), $F_1(r)$, $\Psi(\theta)$, Z(z) we may get a number of magnetic fields with conservative Lorentz Force, one of such field can be obtained by taking

$$\left.\begin{aligned} F(r) &= F_1(r) = r \\ \Psi(\theta) &= \theta \\ Z(z) &= z \\ \beta &= \alpha \end{aligned}\right\} \qquad \text{.....(3.28)}$$

the magnetic field becomes

$$\left.\begin{aligned} H_r &= 0 \\ H_0 &= \alpha . rz \\ H_z &= \frac{\alpha r \theta}{r} = \alpha\theta \end{aligned}\right\} \qquad \text{.....(3.29)}$$

For this magnetic field represented by equation (3.29) it may readily be shown that the Lorentz Force can be represented by the gradient of a scalar quantity ϕ

$$\therefore \quad \phi = \alpha^2 \left[\frac{1}{3} r^3 z^2 + z^2 r^2 + \frac{r^2}{2} + \frac{\theta^2}{2} - \frac{r^2\theta^2}{2} \right] \qquad \text{.....(3.30)}$$

Similarly one more magnetic field with conservative Lorentz Force is given by

$$\left.\begin{aligned} H_r &= 0 \\ H_\theta &= r \\ H_z &= \frac{\delta}{r} \end{aligned}\right\} \qquad \ldots\ldots(3.31)$$

For this magnetic field, ϕ is given by

$$\phi = \frac{\delta^2}{2\lambda^2} - \frac{\gamma^2}{r} \qquad \ldots\ldots(3.32)$$

<u>CASE-II</u> When $H_\theta = 0$, in this case the magnetic field with consevative Lorentz Force is given by

$$\left.\begin{aligned} H_r &= \frac{\alpha_1 \Psi_1(\theta) Z_1(z)}{r} \\ H_\theta &= 0 \\ H_z &= \frac{\beta_1 F_2(r) \Psi_2(\theta)}{r} \end{aligned}\right\} \qquad \ldots\ldots(3.33)$$

and ϕ is given by

$$\begin{aligned} \phi = & -\alpha_1\beta_1\Psi_1(\theta)Z_1'(z)\int \frac{F_2(r)\,dr}{r^2} \\ & -\beta_1^2\{\Psi_2(\theta)\}^2 \int \frac{F_2(r)F_2'(r)}{r^2}\,dr \\ & -\beta_1^2\{\Psi_2(\theta)\}^2 \int \frac{\{F_2(r)\}^2}{r^3}\,dr + \frac{\beta_1^2\{F_2(r)\}^2}{r^2}\int \Psi_2(\theta)\Psi_2'(\theta)\,d\theta \\ & -\alpha_1^2 \frac{Z_1(z)Z_1'(z)}{r}\int\{\Psi_1(\theta)\}^2 d\theta \\ & +\frac{\alpha_1\beta_1 Z_1(z)F_2'(r)}{r}\int \Psi_1(\theta)\Psi_2(\theta)\,d\theta - \frac{\alpha_1\beta_1 Z_1(z)F_2(r)}{r^2}\int \Psi_1(\theta)\Psi_2(\theta)\,d\theta \\ & +\alpha_1^2\frac{\{\Psi_1(\theta)\}^2}{r^2}\int Z_1(z)Z_1'(z)\,dz - \frac{\alpha_1\beta_1\Psi_1(\theta)\Psi_2(\theta)F_2'(r)}{r^2}\int Z_1(z)\,dz \\ & +\frac{\alpha_1\beta_1\Psi_1(\theta)\Psi_2(\theta)F_2(r)}{r^3}\int Z_1(z)\,dz \end{aligned} \qquad \ldots\ldots(3.34)$$

(ii) $$\left.\begin{aligned} H_r &= \frac{\alpha_1 \theta z}{r} \\ H_\theta &= 0 \\ H_z &= \frac{\alpha_1 r \theta}{r} = \alpha_1 \theta \end{aligned}\right\} \qquad \ldots\ldots(3.35)$$

and $$\phi = \alpha_1^2\theta^2 \log r - \frac{\alpha_1^2 \theta^2 z^2}{2r^2} - \alpha_1^2 \theta^2 \log r + \frac{\alpha_1^2\theta^2}{2} - \frac{\alpha_1^2 z\theta^3}{r.3} + \frac{\alpha_1^2 z\theta^3}{3} - \frac{\alpha_1^2 z\theta^3}{3r} + \frac{\alpha_1^2\theta^2 z^2}{2r^2} - \frac{\alpha_1^2\theta^2 z^2}{2r} + \frac{\alpha_1^2\theta^2 z^2}{2r^2}$$

or $$\phi = \frac{\alpha_1^2\theta^2}{2} + \frac{\alpha_1^2 z\theta^3}{3} - \frac{\alpha_1^2\theta^2 z^2}{2r} + \frac{\alpha_1^2\theta^2 z^2}{2r^2} \qquad \ldots\ldots(3.36)$$

(iii) $$\left.\begin{aligned} H_r &= \frac{V_1}{r} \\ H_\theta &= 0 \\ H_z &= \delta_1 \end{aligned}\right\} \qquad \ldots\ldots(3.37)$$

and ϕ = constant. .....(3.38)

<u>CASE-III</u> When $H_z=0$, some magnetic field with conservative Lorentz Force are given by

(i) $$\left.\begin{aligned} H_r &= \frac{\alpha_2 \Psi_3(\theta) Z_2(z)}{r} \\ H_\theta &= \beta_2 F_3(r) Z_3(z) \\ H_z &= 0 \end{aligned}\right\} \qquad \ldots\ldots(3.39)$$

and

$$\phi = \beta_2 Z_3^2(z) \int \{F_3(r)\}^2 dr - \beta_2^2 Z_3^2(z) \int F_3(r) F_3'(r) dr$$

$$- \alpha_2 \beta_2 Z_2(z) Z_3(z) \psi_3'(\theta) \int \frac{F_3(r)}{r^2} dr + \frac{\alpha_2^2 Z_2(z) Z_2'(z)}{r} \int \psi_3^2(\theta) d\theta$$

$$- \alpha_2 \beta_2 Z_2(z) Z_3(z) F_3'(r) \int \psi_3(\theta) d\theta + \frac{\alpha_2^2 \psi_3^2(\theta)}{r^2} \int Z_2(z) Z_2'(z) dz$$

$$+ \beta_2 F_3^2(r) \int Z_3(z) Z_3'(z) dz$$

.....(3.40)

(ii)

$$\left. \begin{aligned} H_r &= \frac{\alpha_2 \theta z}{r} \\ H_\theta &= \alpha_2 rz \\ H_z &= 0 \end{aligned} \right\} \qquad .....(3.41)$$

and

$$\phi = \frac{1}{3} \alpha_2^2 r^3 z^2 - \alpha_2^2 r^2 z^2 - \alpha_2^2 z^2 \log r + \frac{1}{3} \alpha_2^2 \theta^3 z$$

$$+ \frac{\alpha_2^2 \theta^2 z^2}{2 r^2} + \frac{\alpha_2^2}{2} r^2 z^2 \qquad .....(3.42)$$

(iii)

$$\left. \begin{aligned} H_r &= \frac{\nu_2}{r} \\ H_\theta &= \delta_2 \\ H_z &= 0 \end{aligned} \right\} \qquad ......(3.43)$$

and $\phi = \delta_2^2 \log r$ ......(3.44)

In the above discussion all the nine magnetic fields in cylindrical coordinates given by equations (3.26), (3.29), (3.31), (3.33), (3.35), (3.37), (3.39), (3.41), (3.43) have conservative Lorentz Force and these are pressure balanced fields and it is a well known fact

that in such fields the lines of force and current are sittuated in the surface ϕ = Constt.

SOME MAGNETOHYDROSTATIC CONFIGURATIONS

If $\bar{q}$ denotes the fluid velocity of an electrically conducting imcompressible fluid permeated by a magnetic field $\bar{H}$ and ρ the density; the steady state consists of all the solutions of the equation given by (3.18)

$$\rho \operatorname{curl}(\bar{q}) \times \bar{q} - \frac{\mu}{4\pi}(\operatorname{curl}\bar{H}) \times \bar{H} = -\operatorname{grad} p - \rho \operatorname{grad}\left(\tfrac{1}{2}q^2\right) + \rho\bar{F} \quad \text{.....(3.45)}$$

$$\operatorname{Curl}(\bar{q} \times \bar{H}) = 0 \quad \text{.....(3.46)}$$

Here the body force $\bar{F}$ will contain no frictional forces. Let q_r, q_θ & q_z be the components of $\bar{q}$ at any point (r, θ, z) in cylindrical coordinates. These components has already been discussed by [3] for different cases as

(i)

$$\left.\begin{aligned} q_r &= 0 \\ q_\theta &= AUW \\ q_z &= \frac{BU_1V}{r} \end{aligned}\right\} \quad \text{.....(3.47)}$$

(ii)

$$\left.\begin{aligned} v_r &= 0 \\ v_\theta &= A r Z \\ v_z &= A \theta \end{aligned}\right\} \qquad \text{......(3.48)}$$

(iii)

$$\left.\begin{aligned} v_r &= 0 \\ v_\theta &= C_1 \\ v_z &= \frac{D_1}{r} \end{aligned}\right\} \qquad \text{.....(3.49)}$$

(iv)

$$\left.\begin{aligned} v_r &= \frac{A_1 V_1 W_1}{r} \\ v_\theta &= 0 \\ v_z &= \frac{B_1 U_2 V_2}{r} \end{aligned}\right\} \qquad \text{.....(3.50)}$$

(v)

$$\left.\begin{aligned} v_r &= \frac{A_1 \theta Z}{r} \\ v_\theta &= 0 \\ v_z &= A_1 \theta \end{aligned}\right\} \qquad \text{......(3.51)}$$

(vi)

$$\left.\begin{aligned} v_r &= C_1/r \\ v_\theta &= 0 \\ v_z &= D_1 \end{aligned}\right\} \qquad \text{.....(3.52)}$$

(vii)

$$\left.\begin{aligned} v_r &= A_2 V_3 W_2 \\ v_\theta &= B_2 U_3 W_3 \\ v_z &= 0 \end{aligned}\right\} \qquad \text{.......(3.53)}$$

(viii)

$$\left.\begin{aligned} v_r &= \frac{A_2 \theta z}{r} \\ v_\theta &= A_2 r z \\ v_z &= 0 \end{aligned}\right\} \qquad \text{.....(3.54)}$$

(ix)

$$\left.\begin{aligned} v_r &= C_2 / r \\ v_\theta &= D_2 \\ v_z &= 0 \end{aligned}\right\} \qquad \text{.....(3.55)}$$

Where U, U_1, U_2, U_3 are the integrable functions of r; V, V_1, V_2, V_3 are functions of θ and W, W_1, W_2, W_3 the functions of z. Also A, A_1, A_2, B, B_1, B_2, C, C_1, C_2, D, D_1, D_2 are constants which may be determined by boundary conditions.

Now, let us consider the flow of an electrically conducting incompressible fluid with velocity given by equation (3.47) acted upon by the magnetic field given by equation (3.26). If

$$\left.\begin{aligned} &F(r) = U(r) \quad , F_1(r) = U_1(r) \\ &Z(z) = W(z) \quad , \psi(\theta) = V(\theta) \\ &\alpha = \pm\sqrt{\frac{\mu}{4\pi\rho}}\, A \text{ and } \beta = \pm\sqrt{\frac{\mu}{4\pi\rho}}\, .B \end{aligned}\right\} \qquad \text{.....(3.56)}$$

Then the flow (3.47) and magnetic field (3.26) will satisfy the equation (3.46). Also we have

$$\bar{v} = \pm\sqrt{\frac{\mu}{4\pi\rho}}\ \bar{H} \qquad .....(3.57)$$

Wallen [13, 14] has shown that equation (3.57) is always a solution of equation (3.47), if

$\bar{F} = 0$ (no external solution) and if

$$p + \frac{1}{2}\rho v^2 = \text{Constt.} \qquad .....(3.58)$$

Thus the flow of an electrically conducting incompressible fluid with velocity given by equation (3.47) acted upon by the magnetic field given by equation (3.26) constitute a magnetohydrostatic configuration if there are no external forces and then equation (3.56) is satisfied and pressure distribution is given by

$$p = \text{Constant} - \left[\frac{\lambda_1^2 r^2 U^2 W^2 + \lambda_2^2 U_1^2 V^2}{r^2}\right]$$

Similarly we can show that the self-superposable flows and magnetic fields given by equation (3.48) and (3.29); (3.49) and (3.31); (3.50) adn (3.33); (3.51) and (3.35); (3.52) and (3.37); (3.53) and (3.39); (3.54) and (3.41) and (3.55) and (3.43) constitute some more magnetohydrostatic configurations in cylindrical ducts.

Magnetohydrostatic configurations thus found may be of either pressure balanced type or in very special cases force free type [5]. In all these, no secondary flow be possible because the magnetic forces would just balance the centrifugal effects [12].

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CHAPTER IV

CHAPTER-4

SOME MAGNETOHYDRODYNAMIC FLOWS

4.1 In this chapter rotational and irrotational motions have been briefly studied. Some properties of vorticity and irrotational motion and their scope have also been given. This Chapter contains my own contribution in form of two research papers. These papers are :-

1. On the Vorticity of MHD Unsteady Hele-Shaw flow of Non-Newtonian Fluid.

2. On the Vorticity of an Elastico-Viscous Fluid near an accelerated plate.

4.2 Plane Flow :-

The movement of ideal fluids consists of two types differing from each other both physically and mathematically. They are called rotational and irrotational (or potential) flow.

The types of flow will be derived by considering a fluid inplane flow andit a small area in the vicinity of the arbitrary point O (x,y). The velocity at point O is $\bar{v}$ (x,y), point O', a small distance dr from O, has velocity, $\bar{v}' = \bar{v} + d\bar{v}$ with components

$$U' = u + \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy \qquad \text{.........(4.1)}$$

$$V' = v + \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy \quad ....(4.2)$$

For convenience, let

$$a = \frac{\partial u}{\partial x},$$

$$b = \frac{\partial v}{\partial y}$$

$$c = \frac{1}{2} \left(\frac{\partial d}{\partial x} + \frac{\partial u}{\partial y}\right) \quad ....(4.3)$$

and $$\xi = \frac{1}{2} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \quad ......(4.4)$$

The equation (4.1) and (4.2) can be written in the following way

$$U' = u+a \ dx + c \ dy - \xi dy \quad ....(4.5)$$

$$v' = v+b \ dy + c \ dx - \xi dx \quad ....(4.6)$$

To visualize the meaning of the individual terms of this expression consider a rectangular fluid element (Fig.4.1) whose one corner is the point O and whose diagonally opposite corner is O'. Clearly, u and v are the components of translational velocity, i.e., they indicate linear motion of all parts of the elements without change of shape (if $a = b = c = \xi = 0$). In view of (4.3) the terms adx and bdy express the change of velocity along dx and dy, respectively. However, since velocity is distance divided by time, a dx and b dy define the rate of change of length, the stretching rate of the edge of the element along the two principal

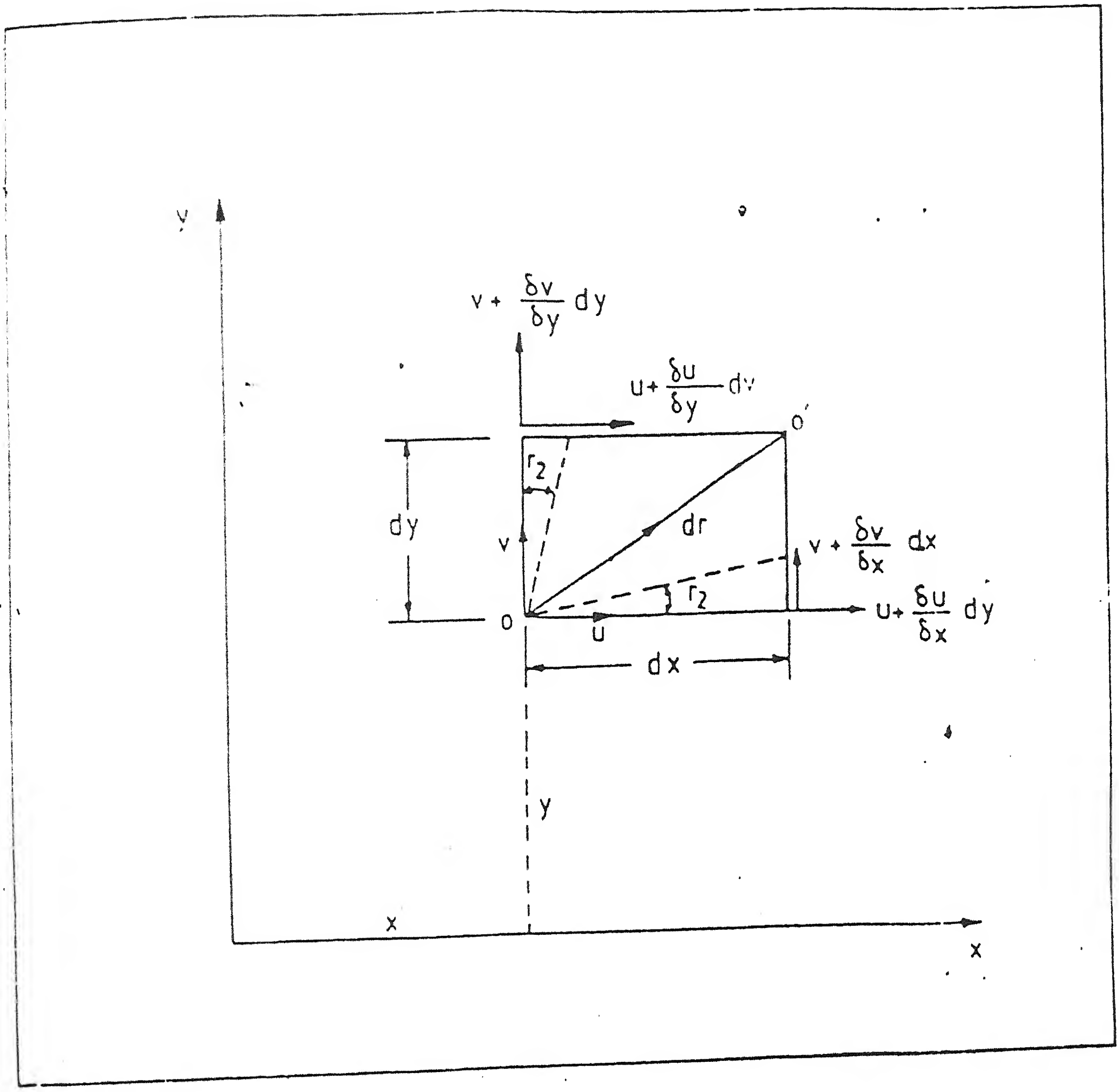

y
$v + \frac{\delta v}{\delta y} dy$
$u + \frac{\delta u}{\delta y} dy$
o'
r_2
dy
v
dr
$v + \frac{\delta v}{\delta x} dx$
r_2
o
u
$u + \frac{\delta u}{\delta x} dy$
dx
y
x
x

Fig. 4.1.

directions,

$$\nu_1 = \frac{\partial v}{\partial x}, \quad \nu_2 = \frac{\partial u}{\partial y} \qquad ....(4.7)$$

so that, with (4.3)

$$\nu_1 + \nu_2 = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = 2a \qquad ....(4.8)$$

defines the change in the angle between the two edges at point O form its original 90°. In other words, the terms containing a, b, and c in equation (4.5) and (4.6) define the deformation of the element.

If the element is considered rigid for a moment, so that a = b = c = 0, then (4.3) requires that $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$ or $\nu_1 = -\nu_2$ because of (4.7). This does not mean that ξ is zero. In fact equations (4.4) and (4.7) show that in this case.

$$2\xi = \nu_1 - \nu_2 = 2\,\nu_1$$

That is, the element is subject to rotation around axis parallel to the z-axis through point O. It should be noted that the angle ν_1 has the dimension 1/time, i.e., it is a change of agnle per unit time or an angular velocity, therefore.

$$\xi = \nu_1 = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

is the angular velocity with which the rigid element moves around the axis through point O.

Although this discussion has been confined to plane motion, it can be extended easily to three dimensions by analogous methods. As a result, the most general motion of fluid is made up of translation, deformation and rotation. In movement in space, the single relation (4.4) is replaced by three relations about mutually perpendicular axes and can beexpressed by

$$\xi = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \qquad \ldots\ldots\ldots(4.9)$$

$$\zeta = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) \qquad \ldots\ldots\ldots\ldots(4.10)$$

$$\eta = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \qquad \ldots\ldots\ldots(4.11)$$

Fluid motions in which one or more of the terms ξ, η and ζ is different from zero is called rotational motion. The vector

$$\bar{\zeta} = i\xi + i\eta + k\zeta \qquad \ldots\ldots\ldots(4.12)$$

is called the vorticity vector : ξ, η, ζ are its components along the three coordinate axes x, y, z while i, j, k, are the unit vectors in the direction of these axes. In vector analysis the vector $2\bar{\zeta}$ that is, the vector whose components are 2ξ, 2η, 2ζ is called the curl of $\bar{v}$ and written curl $\bar{v}$, so that the rotational vector $\bar{\zeta}$ and the velocity vector $\bar{v}$ are connected by the relation,

$$\bar{\zeta} = \frac{1}{2}\,\mathrm{curl}\ \bar{v} \qquad \ldots\ldots\ldots(4.13)$$

The direction of vector $\bar{\zeta}$ is determined by the momentary location of the axis around which the element rotates, and its magnitude is

$$|\bar{\zeta}| = (\xi^2 + \eta^2 + \zeta^2)$$

A flow system is called irrotational if the rotational vector $\bar{\zeta}$ disappears completely. According to (4.9), (4.10) and (4.11) this mean that

$$\left.\begin{aligned} \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} &= 0 \\ \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} &= 0 \\ \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} &= 0 \end{aligned}\right\} \qquad \text{.........(4.14)}$$

and the velocities u, v, w can be expressed as partial derivatives of a function

$\emptyset$ (x, y , z,t)

or

$$\left.\begin{aligned} u &= \frac{\partial \phi}{\partial x} \\ v &= \frac{\partial \phi}{\partial y} \\ w &= \frac{\partial \phi}{\partial z} \end{aligned}\right\} \qquad \text{..........(4.15)}$$

which can really be verified by substitution in equation (4.14). In vector notation, equation (4.15) becomes more simply

$$\bar{v} = \text{grad } \emptyset \qquad \text{.........(4.16)}$$

Since the vectors sum of the three partial differentials is the gradient of $\emptyset$ The function $\emptyset$ is the velocity potential. In steady flow $\emptyset$ is independent of time and a pure point function.

By substituting the values for u, v, and w from (4.15) the equation of continuity for an incompressible fluid.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

one finds that, in irrotational flow

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \nabla^2 \phi = 0 \qquad ....(4.17)$$

Here ∇^2 is the Laplace operator. For this reason the continuity equation (4.17) is often called the Laplace equation. It has been shown that in irrotational flow, the velocity $\bar{v}$ can be derived from a potential. There force such flow is frequently called potential flow.

EQUATION OF MOTION FOR THE VORTICITY

We shall now obtain an equation of motion for an incompressible, inviscid fluid as

$$\frac{\partial u}{\partial t} - \bar{u} \times \bar{\zeta} = \text{grad } \bar{w} \qquad ....(4.18)$$

where

$$\bar{w} = \frac{p}{\rho} + \frac{1}{2} |\bar{u}|^2 + v \qquad ......(4.19)$$

Taking the curl of equation (4.19) we obtain

$$\frac{\partial \bar{\zeta}}{\partial t} - \text{curl } (\bar{u} \times \bar{\zeta}) = 0 \qquad \text{.........(4.20)}$$

This is the required equation of motion for ζ.

MOTION OF VORTEX LINES

It is worthwhile to note that the induction euqation for an infinitely conducting fluid.

$$\frac{\partial \bar{H}}{\partial t} = \text{curl } (\bar{U} \times \bar{H})$$

and the hydrodynamic vorticity equation

$$\frac{\partial \bar{\zeta}}{\partial t} = \text{Curl } (\bar{u} \times \bar{\zeta})$$

obtained when the external body forces per unit mass are conservative and similar. It follows that the vortex line move with the fluid (Helmholtz' theorem) and if the vorticity is initially zero, it is zero for all time.

4.3 THE SOURCE OF VORTICITY IN MOTIONS GENERATED FROM REST.

It can be established that changes in the flux of vorticity across a material surface element take place solely as a consequence of local diffusion of vorticity, by viscosity.

Vorticity flux or circulation can not be created in the interior of the fluid but once there it is spread

by the action of viscosity.

This raises the important question of the ultimate source of vorticity in motions which have been generated from rest in a fluid of uniform density. Initially the vorticity is every where zero, and the motion must remain wholly irrotational unless vorticity diffuses across the surface bounding the fluid. Real fluid motions which can be seen to possess vorticity over atleast part of the field are common so that we are lead to expect that some mechanism exists for the generation of vorticity at the boundary of the fluid.

When the fluid is bounded wholly or partly by a solid, any remaining part of boundary being at infinity where the fluid is at rest, such a mechanism is provided by the no-slip condition. Mechanism for the generation of vorticity do exist at other types of boundary, such as a free surface at which the pressure is constant and the tangential stress is zero, but the case of a solid boundary is by for the most common and it alone need is examined in detail here. An irrotational motion of the fluid is determined completely by the condition of zero flux of mass across each element of the solid boundary, and the unique irrotational motion almost in-evitably has a non-zero tangential component of relative velocity of the fluid at the solid boundary. Thus, the motion that would be generated from rest in the absence of diffusion

of vorticity across the boundary of the fluid is accompanied by a non-zero tangential relative velocity to be zero at each point of the solid boundary, however, small the viscosity may be, the vorticity in this flow is infinite at the boundary. This sheet of infinite vorticity at the boundary is the source from which once viscosity is allowed to act vorticity diffuses into the interior of the fluid.

4.4. IRROTATIONAL MOTION

If the vorticity is zero at all points in a region (except certain special points, called singular points. Where the velocity or the acceleration is theoretically zero or infinite) then the flow in that region is said to be irrotational. Flow in regions, where the vorticity is other than zero is said to be rotational. In practice there may be rotational motion in one part of a flow field and irrotational motion in another part.

The concept of irrotational flow lies behind much of what follows and we may here pause to consider its physical interpretation irrotational flow is flow in which each element of the moving fluid undergoes no net rotation (with respect to chosen coordinate axes) from one instant to another. A well-known example of irrotational motion is that of the carriages of the big

(ferries) wheel in a fair-ground although each carriage follows a circular path as the wheel revolves, it does not rotate with respect to the earth the passengers fortunately remain upright and continue to face in the same direction.

Two examples of fluid flow are, depicted in Fig. 4.2. A small element of fluid as therefore convenience represented by a quadrilateral with axes AB and CD. At (a) the axis AB rotates clockwise as the element moves along, but CD rotates an equal amount anticlockwise so that the net rotation is zero. Although undergoing distortion the element is thus not rotating; the flow is irrotational. At (b), however, both axes rotate in the same direction, there is thus rotation but little distortion.

Rotation of a fluid particle can be caused only by a torgue applied by shear forces on the sides of the particle. Such forces, however, are impossible in an ideal. Inviscid, fluid and so the flow of an ideal fluid is necessarily irrotational.

To illustrate the difference between rotational and irrotational flow by a simple example. Consider some fluid rotating like a rigid body around on axis at right angles to the place of the paper with an angular velocity (Fig.4.3). At radius r_2, the tangential velocity

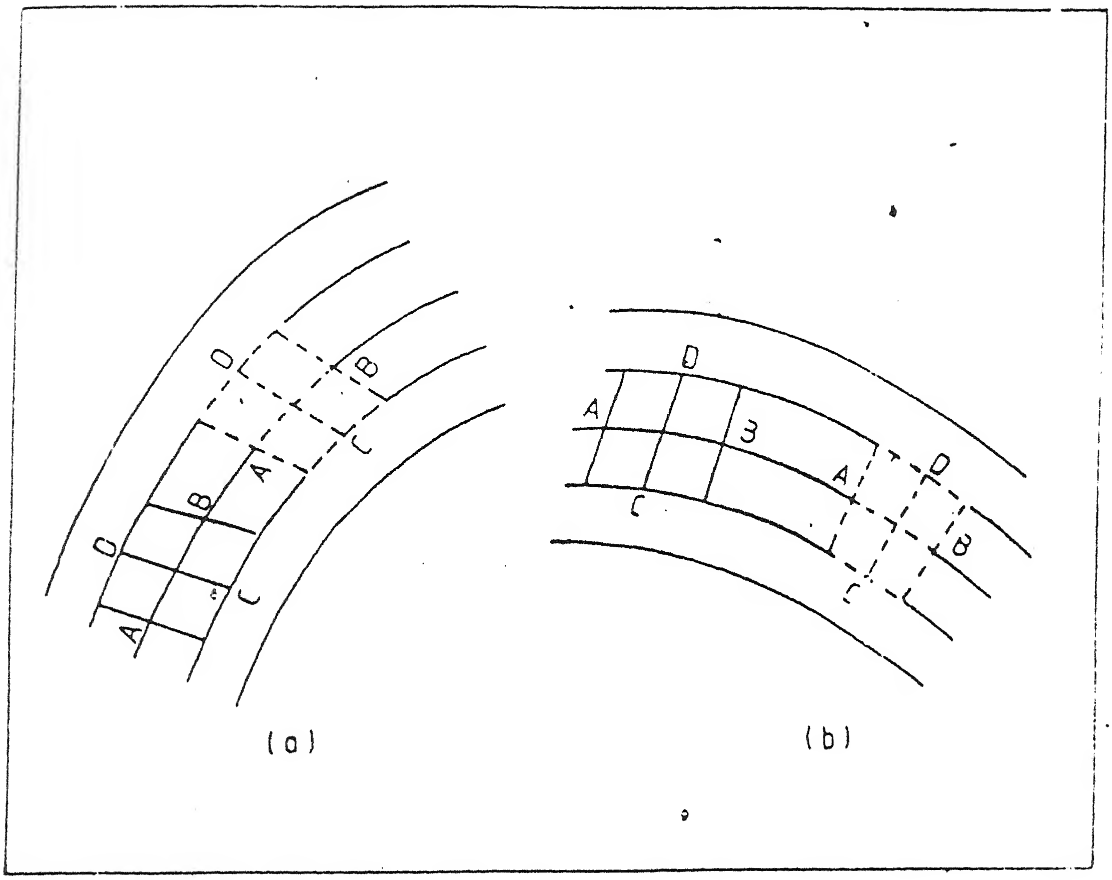
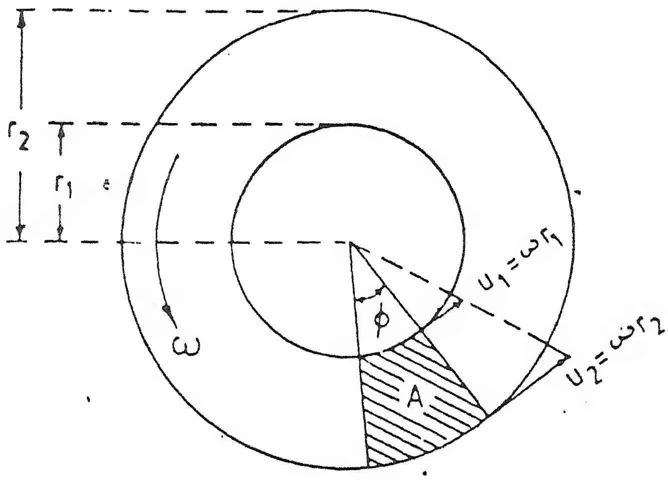
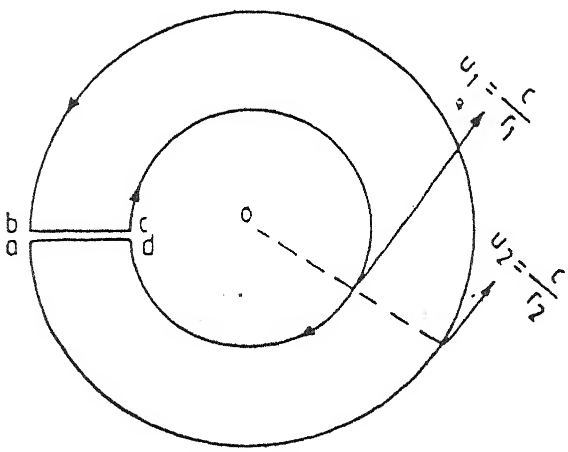


Fig. 4.2



(a)



(b)

Fig. 4.3

$u_2 = wr_2$; at radius r_1 the velocity is $u_1 = wr_1$. The circulation along the boundary of the cross-batched region in Fig. 4.3(a) can be computed easily, since the two radial lines do not contribute to it.

Therefore,

$$\Gamma = u_2 r_2 \emptyset - u_1 r_1 \emptyset$$

$$= \emptyset w (r_2^2 - r_1^2)$$

The area of the region is

$$A = \int_{r_1}^{r_2} (r\emptyset)\, dr$$

$$= \frac{\emptyset}{2} (r_2^2 - r_1^2)$$

So that

$$\Gamma = 2\omega A$$

In other Words, the circulation is equal to the product of the angular velocity and the enclosed area A. Obvisiously, and circulation is valid for any arbitrary circular sector, even an infinitesimally small one, since the angular velocity is the same everywhere. The arguments regarding the rotational vector show that its component must be identical with (of course, and are zero in this case), so that all particle, in the fluid have rotation and the rotational vector has a uniform value throughout.

To contrast, consider a flow system in which the stream lines are also concentric circles around O but in which the tangential velocity is inversely proportional to the distance from the origin or equal to $\frac{C}{r}$ where C is a constant, if the circulation is computed around the area similar to that in Fig.4.3, then

$$\Gamma = u_2 r_2 \phi - u_1 r_1 \phi = 0$$

i.e., the region is irrotational. It is easy to show that this is true for any arbitrary line a b c d which does not include the centre of rotation O, Fig.4.3(b). On other hand, the circulation of a closed circle around O is

$$\Gamma' = 2\pi r . \frac{C}{r} = 2\pi C = \text{constant}$$

This circulation is independent of r and gives physical meaning to the constant C. Since is different from zero, rotation must exist at point O. Therefore, point O is a singular point at which u is infinitely large.

Now we will consider the properties of a velocity field $\bar{v}$ satisfying the equations

$$\bar{\nabla} . \bar{v} = 0, \quad \bar{\nabla} \times \bar{v} = 0 \qquad ....(4.21)$$

We can see that most fluids behave under a wide range of flow conditions, as if they were nearly

incompressible, and also that flow fields with the seemingly restrictive property of zero vorticity over large parts of the field area, for dynamical reasons, remarkably common, Study of irrotational solenoidal vector fields therefore has great practical claue in fluid mechanics.

The implicity of the equation (4.21) has also made possible extensive mathematical of powerful analytical techniques.

In a fluid in which the instantaneous distribution of velocity is $\bar{v}$ $(\bar{x})$ material elements are being subjected to translation and a pure straining motion without change of volume and without superposed rotation

Since $\bar{\nabla} \times \bar{v}$ is zero at all points of the fluid, Stoke's theorem show that

$$\oint \bar{v} \cdot d\bar{x} = 0 \qquad \text{..........(4.22)}$$

for all reducible closed curves lying within the fluid, because it is always possible to find an open surface bounded by any such reducible curve and lying entirely in the fluid. If O and P are two points in a connected region of fluid and C_1 and C_2 are two different curves joining O and P in such a way that the two together from a reducible closed curve lying entirely in the fluid,

we see from equation (4.22) that

$$\int_{C_1} \bar{v} \, d\bar{x} = \int_{C_2} \bar{v} \, d\bar{x}$$

The line integral of $\bar{v}$ over a curve joining O to P and lying within the fluid thus has the same value for all members of a set of paths of which any two make a reducible closed curve and depends only on the position vector $\bar{x}_o$ and $\bar{x}$ of O and P respectively. It is therefore possible to define a function ϕ (x) such that

$$\phi(\bar{x}) = \phi(\bar{x}_o) + \int_O^P \bar{v} \, d\bar{x} \qquad \ldots\ldots\ldots\ldots(4.23)$$

in which the integral is taken over one of the paths in the set mentioned. The vector gradient of $(\bar{x})$ is found by varying the position of P, giving

$$\nabla \phi(\bar{x}) = \bar{V}(\bar{x}) \qquad \ldots\ldots\ldots\ldots(4.24)$$

is termed as the velocity potential for the field $\bar{v}$ (although there is no question here of an interpretation of ϕ as a potential energy function). It is customary to leave the position $\bar{x}_o$ unspecified, since the difference between the values of corresponding to two different choices of $\bar{x}_o$ is independent of $\bar{x}$ and so without effect on

$$\nabla \phi(\bar{x})$$

The introduction of the function ϕ by means of the relation (4.24) ensures that the equation $\overline{\nabla} \times \overline{v} = 0$ is satisfied identically, and the three unknown scalar components of $\overline{v}$ are thereby determined by a single unknown scalar function ϕ. The first of the equation (4.21) then requires

$$\nabla^2 \phi = 0 \qquad \text{........(4.25)}$$

at all points of the fluid. This equation for ϕ, known as Laplace's equation, appears in many branches of mathematical physics, and many general results about functions satisfying the equation are known. The linearity of the equation is note-worthy and accounts for the relative simplicity of analysis of irrotational solenoidal flow, the dynamical equations governing the change of the velocity distribution in a fluid from one instant to the next are in general nonlinear but in the particular case of irrotational solenoidal flow the constraints on the velocity distribution are so strong as to require the spatial distribution of v to satisfy the simple linear equations (4.24) and (4.25) independently to temporal changes.

4.5 This Section contains my two research papers.

PAPER-I

ON THE VORTICITY OF MHD UNSTEADY HELE-SHAW FLOW OF NON-NEWTONIAN FLUID

ABSTRACT

In this paper an attempt has been made to study the vorticity of unsteady flow of non-Newtonian fluid of Kuvshinki's type under the influence of uniform transverse magnetic field, assuming the pressure gradient to be proportional to exp (-nt). The effect of magnetic field parameter (M) and relaxation time (T) on the vorticity of flow have been studied numerically and graphically.

INTRODUCTION

The steady Hele-Shaw flows have been discussed by Lamp [5], Reigels [7], Thompson [10], Lee and Fung [6] and Buckmaster [1]. In most of these investigations, the pressure gradient was assumed to be consant. Swaminathan [9] has investigated the unsteady Hele-Shaw flow of viscous, incompressible fluid taking the pressure gradient to be a function of time. Gupta et al. [2, 3] studied the same problem for viscoelastic fluid. The purpose of this paper is to obtain an expression for vorticity of unsteady Hele-Shaw flow of non-Newtonian fluid under the influence of transverse magnetic field.

The effect of various parameters viz., magnetic field parameter (M) and relaxation time (T) on the vorticity of flow have been discussed numerically and graphically.

FORMULATION AND SOLUTION OF THE PROBLEM

Let us consider the flow of a non-Newtonian fluid confirned between two parallel plates Z = $\pm$ h past a circular cylinder $x^2 + y^2 = a^2$, $h \leq Z \leq h$ under the influence of uniform transverse magnetic fluid.

The non-dimensional equations governing the motion of non-Newtonian fluid of Kuvshinski type in the Hele-Shaw cell (Gupta et al. [2, 3] after applying the uniform transverse magnetic field of intensity Ho, under the assumption that the effect of the induced magnetic field and electric field produced by the motion of the electrically conducting fluid is negligible and no external electric field is applied, are given by,

$$\left(1+T\frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = -\left(1+T\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial z^2} - M\left(1+T\frac{\partial}{\partial t}\right)u \quad \ldots \quad (4.26)$$

$$\left(1+T\frac{\partial}{\partial t}\right)\frac{\partial v}{\partial t} = -\left(1+T\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial z^2} - M\left(1+T\frac{\partial}{\partial t}\right)v \quad \ldots \quad (4.27)$$

$$-\left(1+T\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial z} = 0 \quad \ldots \quad (4.28)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \ldots \quad (4.29)$$

Where

$$t^* = \frac{t\nu}{a^2}, \quad u^* = \frac{u}{U_0}, \quad v^* = \frac{v}{U_0}$$

$$p^* = \frac{a p}{\nu \rho U_0}, \quad x^* = \frac{x}{a}, \quad y^* = \frac{y}{a}$$

$$z^* = \frac{z}{a}, \quad h^* = \frac{h}{a},$$

T = $\left(\frac{\lambda_0 \nu}{a^2}\right)$ is the relaxation time parameter.

μ_e the magnetic permeability.

σ the electrical conductivity.

The boundary conditions are,

$$u=0, v=0, \quad \text{on} \quad Z = \pm h \qquad ....(4.30)$$

Using (4), the equation (1) and (2) yield

$$\left(1+T\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}\right) = 0 \qquad ....(4.31)$$

Equation (4.28) shows that p is independent of Z. Therefore, p is a function of x, y and t.

Let

$$u = f(t,z)\frac{\partial \phi}{\partial x} \quad \text{and} \quad v = f(t,z)\frac{\partial \phi}{\partial y} \qquad ....(4.32)$$

Where ϕ is some function of x & y.

Substituting the values of u and v from (4.32) in equation (4.29), we get,

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0 \qquad .....(4.33)$$

Again substituting the values of u and v in (4.26) and (4.27) and then integrating, we get,

$$\left(1+T\frac{\partial}{\partial t}\right)p = \left[f_{zz} - Tf_{tt} - (1+MT)f_t - Mf\right]\phi + g(t) \qquad .....(4.34)$$

Where g(t) is an arbitrary function of time.

Let the pressure gradient be proportional to exp(-nt). In equation (4.34) we assume that

$$f_{zz} - Tf_{tt} - (1+MT)f_t = - Ae^{-nt} \qquad ......(4.35)$$

Where N $\in$ I + and A is a given constant.

Let $$f\,(t,z) = e^{-nt}\, F\,(Z) \qquad ....(4.36)$$

Solving (4.35) after using (4.36), under the corresponding boundary conditions, the function, f(t,z) is given as

$$f\,(t,z) = -\frac{Ae^{-nt}}{a_1^2}\left(1-\frac{\cos a_1 z}{\cos a_1 h}\right) \qquad ....(4.37)$$

Where $$a_1^2 = n\,(1+MT) - (M+n^2T) \qquad ....(4.38)$$

The function ϕ (x, y) can be obtained by solving (4.33) subject to the condition

$$u \cos\theta + v \sin\theta = 0 \quad \text{on } r = a \text{ or } \frac{\partial\phi}{\partial r} = 0 \text{ on } r = a \qquad .(4.39)$$

Where $$x = r\cos\theta\ ,\ y = r\sin\theta$$

and $$\frac{\partial\phi}{\partial x} \to 1\ ,\ \frac{\partial\phi}{\partial y} \to 0 \quad \text{as } |x|, |y| \to \infty$$

Hence $$\phi\,(x,\,y) = \left(r + \frac{a^2}{r}\right)\cos\theta \qquad ....(4.40)$$

From equation (4.32), (4.37) and (4.40), the dimensionless velocity components are given by

$$\left.\begin{aligned} u &= -\frac{A\,e^{-nt}}{a_1^2}\left(1-\frac{\cos a_1 z}{\cos a_1 h}\right)\left[1-\frac{(x^2-y^2)}{(x^2+y^2)^2}\right] \\ v &= \frac{A\,e^{-nt}}{a_1^2}\left(1-\frac{\cos a_1 z}{\cos a_1 h}\right)\left[\frac{2xy}{(x^2+y^2)^2}\right] \end{aligned}\right\} \quad \ldots\ldots\ldots (4.41)$$

The vorticity of flow (16) is given by

$$|\vec{\zeta}| = \frac{A_1 e^{-nt}}{a_1}\left(\frac{\sin a_1 z}{\cos a_1 h}\right)\left[\frac{4x^2y^2}{(x^2+y^2)^4}+\left\{1-\frac{(x^2-y^2)}{(x^2+y^2)^2}\right\}^2\right]^{1/2} \ldots\ldots\ldots (4.42)$$

The study the effect of magnetic field parameter (M) and relaxation time (T) on the vorticity of flow, numerical calculations have been carried out and vorticity profiles have been obtained for various values of M and T.

TABLE 1

$n = 1$, $h = 1$, $A = 10$, $M = 0$, $t = 0$

T	ξ \ Z	0.0	0.1	0.2	0.3	0.4	0.5
0	ξ	0	1.8477	3.6770	5.4695	7.2074	8.8732
0.2	ξ	0	1.5954	3.1780	4.7352	6.2546	7.7240
0.4	ξ	0	1.3977	2.7872	4.1598	5.5076	6.8254

TABLE 2

n=1, h=1, A=10, M=0.5, t=0

T	$\xi \backslash$ Z	0.0	0.1	0.2	0.3	0.4	0.5
0	ξ	0	1.3143	2.6219	3.9165	5.1915	6.4406
0.2	ξ	0	1.2389	2.4730	3.6971	4.9065	6.0962
0.4	ξ	0	1.1702	2.3380	3.4983	4.6480	5.7838

TABLE 3

n=1, h=1, A=10, M=0, t=0.2

T	$\xi \backslash$ Z	0.0	0.1	0.2	0.3	0.4	0.5
0	ξ	0	1.5126	3.0104	4.4781	5.9009	7.2648
0.2	ξ	0	1.3062	2.6019	3.8769	5.12089	6.3239
0.4	ξ	0	1.1444	2.2819	3.4061	4.5092	5.5856

TABLE 4

n=1, h=1, A=10, M=0, t=0.2

T	$\xi \backslash$ Z	0.0	0.1	0.2	0.3	0.4	0.5
0	ξ	0.000	1.0760	2.1467	3.2066	4.2505	5.2732
0.2	ξ	0.000	1.01439	2.0247	3.0269	4.0171	4.9912
0.4	ξ	0.000	0.9585	1.9142	2.8641	3.8054	4.7354

DISCUSSION

The effect of magnetic field parameter (M) and the relaxation time (T) on the resultant vorticity ζ are shown graphically and in tabular form. From fig. 1 and 2 and tables 1, 2, 3 and 4 it is clear that at Z=0, vorticity of the flow is zero. But as was shown by Gupta, Sisodia and Sharma (3) the velocity components at Z=0 have maximum value. Hence we can easily conclude that a pure irrotation motion exists in between two planes at Z=0. As Z increases the vorticity also increases and at Z=0.5 its value becomes maximum. But as the velocity components become zero at Z=0.5, hence we have pure vortex motion there.

As relaxation time increases, the vorticity value is depressed and its rate of increase also comes down. Hence the role of relaxation time is to decrease the vorticity of the flow. Although the increasing tendency of vorticity does not change but its rate of increase comes down.

As the magnetic field is induced, the rate of increase is further supressed. Although at h=0.5, we get pure vortex motion but amount of rotation becomes lesser and lesser by the induction of magnetic field and relaxation time.

The increase in time also suppresses the vorticity.

Thus, the induction of magnetic field, relaxation time and time all combindly try to decrease the rate of increase of vorticity and region of irrotationallity at the central time and the region of maximum vorticity along the planes can be found in the flow.

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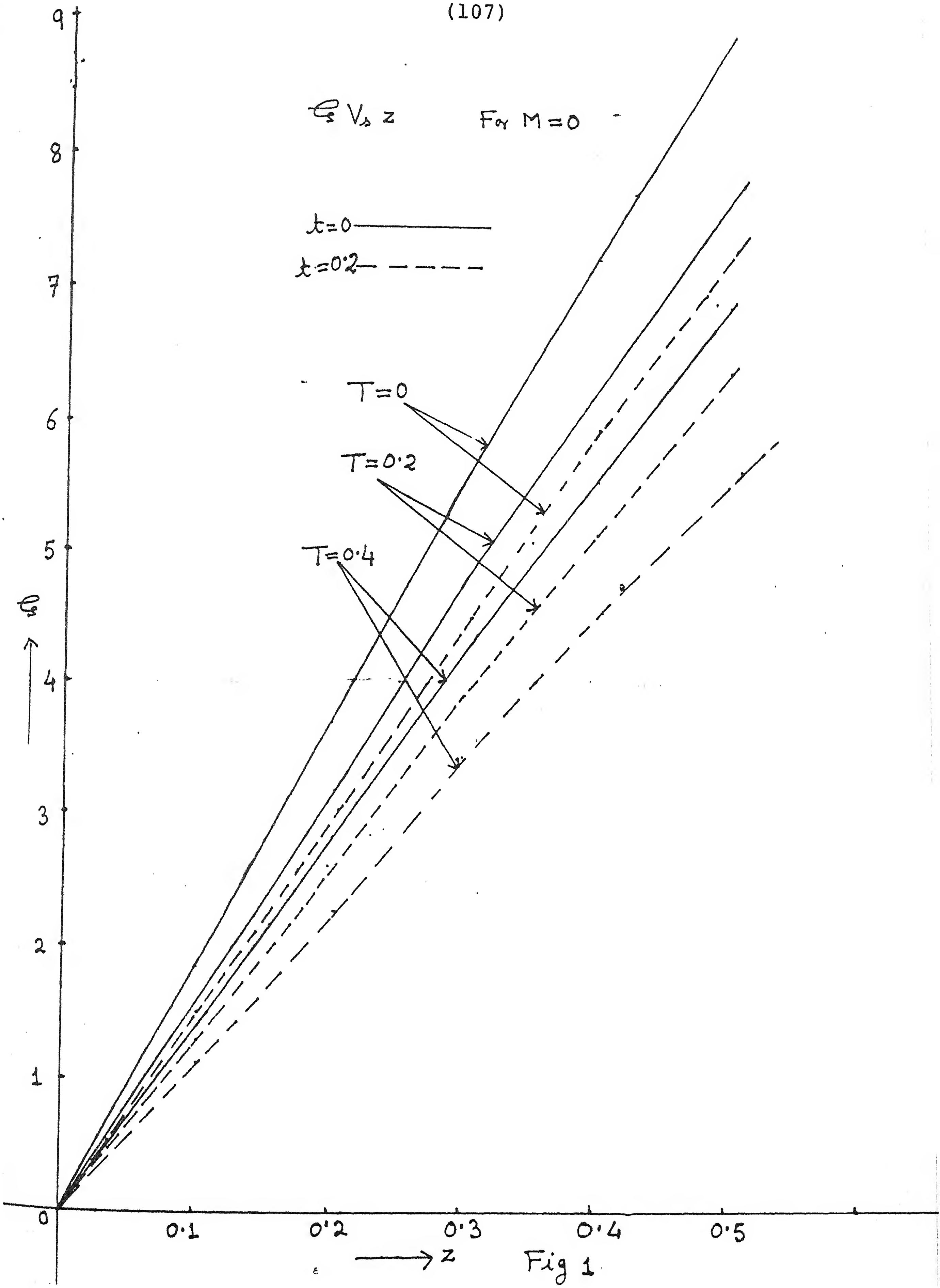
θ_s Vs z For M = 0
t = 0
t = 0·2
T = 0
T = 0·2
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Fig 1

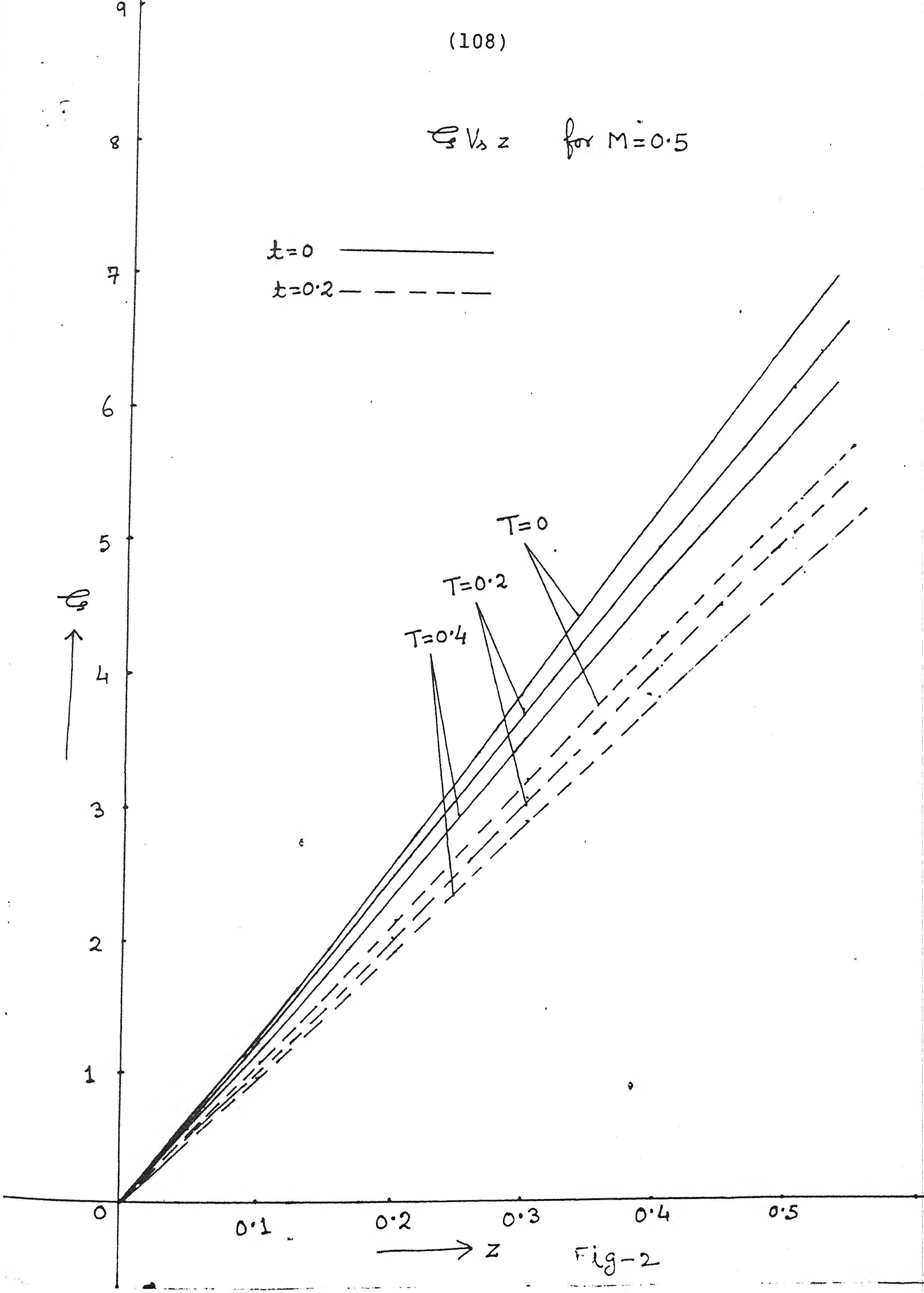

Fig-2

PAPER - II

4.6 ON THE VORTICITY OF FLOW OF AN ELASTICO-VISCOUS FLUID NEAR AN ACCELERATED PLATE

ABSTRACT

In this paper an attempt has been made to study the vorticity of flow of an elastico-viscous fluid in case of Walters liquid β past a uniformly accelerated plate. The effect of elasticity parameter K on the vorticity of flow have been studied numerically and graphically.

INTRODUCTION

The study of the flow behaviour of Non-Newtonian fluid is receiving the attention of research workers in technological field. It has manu practical applications. The flow of an elastico-viscous fluid with short memory, past and impulsively started plate was studied by soundalgekar [8], Haldavnekar [4] has studied the flow of an electro-viscous fluid (walters liquid β) near an accelerated plate. In this paper we have obtained an expression for the vorticity of flow of an elastico-viscous fluid near an accelerated plate. Effect of elasticity parameter on vorticity of flow have been studied numerically and graphically.

FORMULATION AND SOLUTION OF THE PROBLEM

Let us suppose that an infinite plate is uniformaly accelerated in the X'-axis which is taken along the plate. The Y'-axis is taken normal to the plate. The fluid and the plate are assumed to be stationary at $t' < 0$, where t' is the time. The plate is accelerated at $t'=0$. The equation governing the motion is

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} - k' \frac{\partial^2 u'}{\partial y'^2 \partial t'} \qquad \ldots\ldots\ldots(4.43)$$

Where $\nu = \frac{\eta_o}{\rho'}$ and $k' = \frac{k_o}{\rho'}$

The boundary conditions are

$$\left.\begin{array}{lll} u' = 0 & \text{at} & t' \leq 0 \\ u = At'^n & \text{at} & y' = 0 \\ u' = 0 & \text{at} & y' \to \infty \end{array}\right\} \qquad \ldots\ldots\ldots\ldots(4.44)$$

For such fluids $K_o \ll 1$,

To solve the equation (i), let us assume that

$$u' = u_o + K'u_1 \qquad \ldots\ldots\ldots\ldots(4.45)$$

putting the value of u' from (4.45) in equation (4.43) & (4.44) and equating the coefficients of different powers of K' neglecting those of K'^2, we get

$$\frac{\partial u_o}{\partial t'} = \nu \frac{\partial^2 u_o}{\partial y'^2} \qquad \ldots\ldots\ldots(4.46)$$

$$\frac{\partial u_1'}{\partial t'} = \nu \frac{\partial^2 u_1}{\partial y'^2} - \frac{\partial^2 u_0}{\partial y'^2 \partial t'} \qquad \text{..........(4.47)}$$

$$\left.\begin{array}{lll} \text{and } u_o = At^n & u_1=0 & \text{at } y'=0 \\ u_o = 0 & u_1=0 & \text{at } y' \to \infty \end{array}\right\} \qquad \text{.......(4.48)}$$

Applying Laplace transform technique, the solution of equation (4.46) & (4.47) was given by [4] as,

$$u_o = A\left[(t' + \frac{y'^2}{2\nu})\,\mathrm{erfc}\left(\frac{y'}{2\sqrt{\nu t'}}\right) - y'\left(\frac{t'}{\pi\nu}\right)^{1/2} e^{-y'^2/4\nu t}\right]$$

$$u_1 = -\frac{Ay'}{2\nu^2}\left(\frac{\nu}{\pi t'}\right)^{1/2} e^{-y'^2/4\nu t'} \qquad \text{..........(4.49)}$$

Introducting the non-dimensional quantities,

$$\eta = \frac{y}{\sqrt{\nu t'}}, \quad K = \frac{K'}{\nu t'}, \quad u = \frac{u'}{At'}$$

we have from equation (4.45) adn (4.49),

$$u = \left(1 + \frac{\eta^2}{2}\right)\mathrm{erfc}\left(\frac{\eta}{2}\right) - \frac{\eta}{\sqrt{\pi}} e^{-\eta^2/4} - \frac{K\eta}{2\sqrt{\pi}} e^{-\eta^2/4} \qquad \text{.........(4.50)}$$

The vorticity of flow (4.50) is given by

$$\zeta = \eta\,\mathrm{erfc}\left(\frac{\eta}{2}\right) + \frac{e^{-\eta^2/4}}{4\sqrt{\pi}}\left[\eta^3 + \eta^2(K+2) + 8\eta - 2K - 4\right] \qquad \text{....(4.51)}$$

The study the effect of elasticity parameter K on the vorticity of flow (4.51) numerical calculations have been carried out and vorticity profiles have been

obtained for various values of K

K	$\zeta \backslash \eta$	0	0.5	1.0	1.5	2.0	2.5	3.0
0	ξ	-0.5642	0.2834	1.0774	1.6157	1.7703	1.5685	1.1697
0.5	ξ	-0.7052	+0.1675	1.02251	1.6257	1.8222	1.6313	1.2163
1.0	ξ	-0.8463	0.0449	1.0774	1.6358	1.8741	1.6942	1.2683

CONCLUSIONS :

The variation of vorticity, and the influence of elastic parameter K on it has been depicted in the firgure 3. From the vorticity profile and the numerical calculations, we observe that

(i) The vorticity increase with η and somewhere near the plate $(0.5 > \eta > 0)$ region of zero vorticity is observed.

(ii) The vorticity increase upto $\eta = 2$ and then it begins to decrease.

(iii) As the value of K increases, the value of vorticity also increases.

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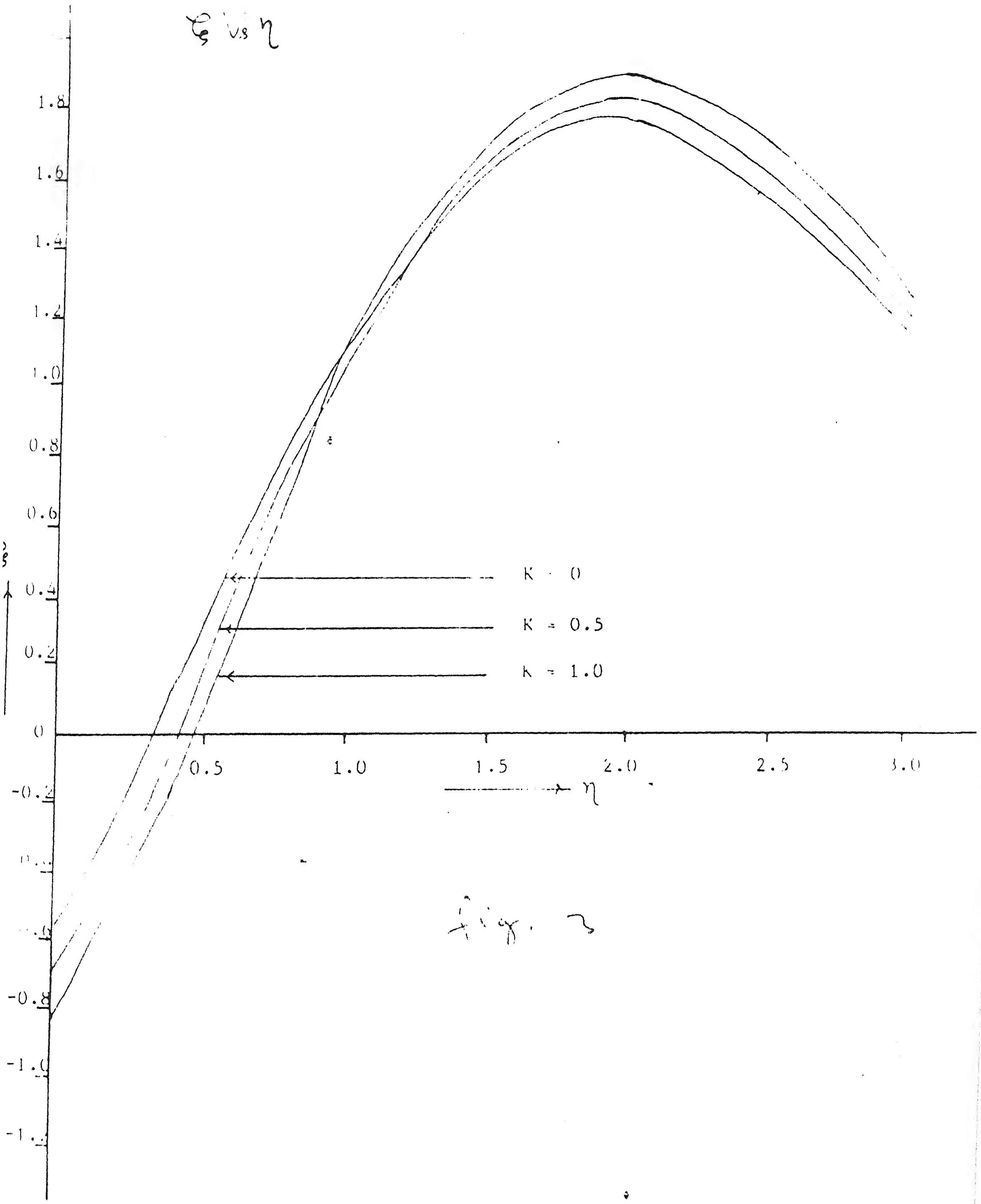

ξ vs η
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
-0.2
-0.8
-1.0
ξ
K = 0
K = 0.5
K = 1.0
0.5
1.0
1.5
2.0
2.5
3.0
η

fig. 3

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CHAPTER V

CHAPTER - 5

VORICITY IN POROUS MEDIUM

5.1 In this chapter fluid motion in porous medium has been considered. The effect of permeability of porous medium, time and electrical conductivity has been studied. This chapter contain two research papers :

1. On the Vorticity of flow of a viscous fluid in a porous medium due to an oscillatiny flat plate.
2. On the Vorticity of Hydromegnetic Visco-elastic fluid near oscillating porous flat plate.

5.2 MHD FLOW IN POROUS MEDIA

The study of physics of flow through porous media has become the basis for many scientific and engineering applications. The study of MHD flows through porous media is receiving considerable attention of researchers owing to its importance in many scientific and engineering fields, particularly in power generation. MHD pumps are already in use in chemical engineering technology for pumping electrically conducting fluids in the Atomic Energy centres. Such type of flows areimportant in petroleum technology to study the movement of gas, oil and water through the oil reservoirs and in chemical engineering for filteration and purification processes. Beyond this, the study is widely applicable in soil mechanics, water

purification, ceramic engineering and powder metallurgy.

The study of MHD flow through porous media is also of great interest in geophysical to study the interaction of the geomagnetic field with the fluid in the geothermal region. Water in the geothermal region is an electrically conducting liquid because of high temperature. With the fuel crisis deepening all over the developed world, attention is turning to the utilization of the enormous power beneath the earth's crust in the geothermal region.

Another potential geophysical application may be in the exploration of geopressured reservoirs. In these reservoirs, water at elevated temperature exists at enormously high pressure because of the weight of the overlying rock and the geomagnetic field. The upflowing water from geopressured wells can run hydraulic turbines to produce electricity, while the heat in the water can simultaneously be extracted to run steam turbines, again producing electricity.

Ahmadi and Manvi [1] derived a general equation of motion of fluid flow through porous media and applied to some basic flow problems. Gulab Ram and Mishra [5] studied MHD flow of conducting fluid through porous media. Varshney [30] studied unsteady MHD flow

through a porous medium in a circular pipe. Gupta [6] studied unsteady MHD flow through porous media in a channel of cross-section circular sector while Kumar, Gupta and Varshney [7] studied MHD channel flow through a porous medium.

In the present thesis I have also tried to study two MHD flows through porous medium and this work has been included as two research papers.

5.3 VORICITY IN MAGNETOHYDRODYNAMIC FLOW

The workers dealing with MHD have paid less attention on the property of vorticity of the flows. Recently, Mittal [11,12,13,14,15] and Mittal et al [16,17, 19,20, 21,22,23] have studied the vorticity of some MHD flows. Here,in this section, the voriticity of MHD flow on or around a vertical porous plate under different magnetic field has been studied. The effect of the applied magnetic field on the vorticity of the flow has been shown by 'vorticity profiles' under different contions. Attempts have been made to find the conditions of irrotationality of the motion which makes the study of flow simple. Also, in the region of irrotationality the energy losses minimize.

Phenomenon of sueprposability and self superposability having been proved to be the most applicable techniques in solving the hydrodynamic and magnetohydrodynamic problems have been frequently used

here in bringing about some very important conclusions about the MHD duct flow.

In the following pages, I have given two research papers, which are my own contributions and deal with the study of vorticity of different MHD flows under different conditions.

5.4 ON THE VORTICITY OF FLOW OF A VISCOUS FLUID IN A POROUS MEDIUM DUE TO AN OSCILLATING FLAT PLATE

ABSTRACT

In this paper an attempt has been made to study the vorticity of unsteady flow of a viscous fluid in a porous medium due to an oscillating flat plate. The effect of permeaility of the porous medium and time on the vorticity of flow have been studied numerically and graphically by plotting the vorticity profiles and results have been discussed critically.

INTRODUCTION

Fluid flow through porous media is of considerable importance in a wide range of disciplines in science and technology, e.g., in soil mechanics, ground water hydrology, petroleum engineering, water purification, industrial filtration, ceramic engineering etc.

The theory of laminar flow through, homogeneous media is based on an experiment originally conducted by Darcy [3], Muskat [5] discussed the flow through porous

media in connection with filtration. Ahmadi & Manvi [1] gave a general equation governing the motion of a viscous fluid through porous media. The flow of a viscous fluid about an infinite flat wall executing harmonic oscillations parallel to itself was first studied by stokes [11]. Rukmangadachari [10] has obtained an expression for velocity of flow of a viscous fluid in a porous medium due to an oscillating flat plate.

In this paper we proposed to study the vorticity of the flow of an incompressible viscous fluid through a porous medium due to the oscillations of a flat plate in its own plane. Numberical calculations have been carried out and vorticity profiles have been obtained for values of k, the permeability of the porous medium and time.

FORMULATION AND SOLUTION OF THE PROBLEM

The equation governing the motion of an incompressible viscous fluid permeating a porous medium of premeability k by [1] are

$$\nabla . \vec{u} = 0 \tag{5.1}$$

$$\rho \frac{\partial u}{\partial t} = - \nabla p - \frac{\mu \vec{u}}{k} + \mu \nabla^2 \vec{u} \tag{5.2}$$

where $\vec{u}$ is the velocity vector, ρ is the density , μ is the coefficient of viscosity and p is the pressure.

Let us consider an infinite flat oscillating in its own plane with velocity $u_0 e^{i.\sigma.t}$ in an infinite expanse expanse of an incompressible viscous fluid

permeating a porous medium of permeability k. Take the x-axis along the plate and the y-axis normal to the plate so that the plate is defined by y=0. For rectilinear flow, the non-zero component of velocity, u is a function of y and t only. The equation (5.1) is identically satisfied and equation (5.2) becomes

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} - \frac{\mu}{k} u + \mu \frac{\partial^2 u}{\partial y^2} \quad \ldots \quad (5.3)$$

In this case $u = u(y) e^{i\sigma t}$ and equation (5.3) reduces to

$$\frac{\partial^2 u}{\partial y^2} - m^2 u = 0 \quad \ldots \quad (5.4)$$

where

$$m^2 = \frac{1}{k} + \frac{i\sigma}{\nu} \quad \ldots \quad (5.5)$$

The boundary conditions are

$$\left.\begin{array}{ll} u = u_0 & \text{for } y = 0 \\ u \to 0 & \text{as } y \to \infty \end{array}\right\} \quad \ldots \quad (5.6)$$

The Solution of equation (5.4) according to [1] is

$$u = u_0 \exp\left[-y\sqrt{\left(\frac{1}{k} + \frac{i\sigma}{\nu}\right)}\right] e^{i\sigma t} \quad \ldots \quad (5.7)$$

In real terms the velocity is given by

$$u = u_0 \exp(-y\sqrt{R}\cos \phi/_2)\cos(\sigma t - y\sqrt{R}\sin \phi/_2) \ldots (5.8)$$

Where

$$\left.\begin{array}{l} R = \left(\frac{1}{k^2} + \frac{\sigma^2}{\nu^2}\right)^{1/2} \\ \quad = \frac{1}{k}\left[1 + \left(\frac{k\sigma}{\nu}\right)^2\right]^{1/2} \\ \phi = \tan^{-1} \frac{k\sigma}{\nu} \end{array}\right\} \quad \ldots \ldots (5.9)$$

The vorticity of flow (5.8) is given by

$$\zeta = u_0\sqrt{R}\exp(-y\sqrt{R}\cos \phi/_2)\cos(\sigma t + \frac{\phi}{2} - y\sqrt{R}\sin \phi/_2) \quad \ldots . (5.10)$$

Let $\frac{\zeta}{u_0\sqrt{R}} = \zeta^*$, $\frac{y}{\sqrt{k}} = y^*$ & $\bar{k} = \frac{k\sigma}{\nu}$, $\sigma . t = \bar{t}$ ...

The equation (5.10) reduces to

$$\zeta^* = \exp\left[-y^*\{1+\bar{k}^2\}^{1/4}\cos\frac{\phi}{2}\right]\cos\left[\bar{t} + \frac{\phi}{2} - y^*(1+\bar{k}^2)^{1/4}\sin\frac{\phi}{2}\right] \ldots (5.1$$

putting $\sqrt{1+\bar{k}^2} = r$

We have

$$\zeta^* = \exp\left[-y^* r^{1/2}\cos\frac{\phi}{2}\right]\cos\left[\bar{t} + \frac{\phi}{2} - y^* r^{1/2}\sin\frac{\phi}{2}\right]$$

TABLE 1

$\bar{t} = 0.4$

$\bar{k}$	$\xi^* \backslash \eta^*$	0.0	0.2	0.4	0.6	0.8	1.0	1.2
0.2	ξ^*	0.9944	0.8134	0.6653	0.5442	0.4451	0.3641	0.2978
0.6	ξ^*	0.9618	0.7813	0.6346	0.5156	0.4188	0.3402	0.2758
1.0	ξ^*	0.9211	0.7399	0.5943	0.4863	0.3835	0.3081	0.2474

TABLE 2

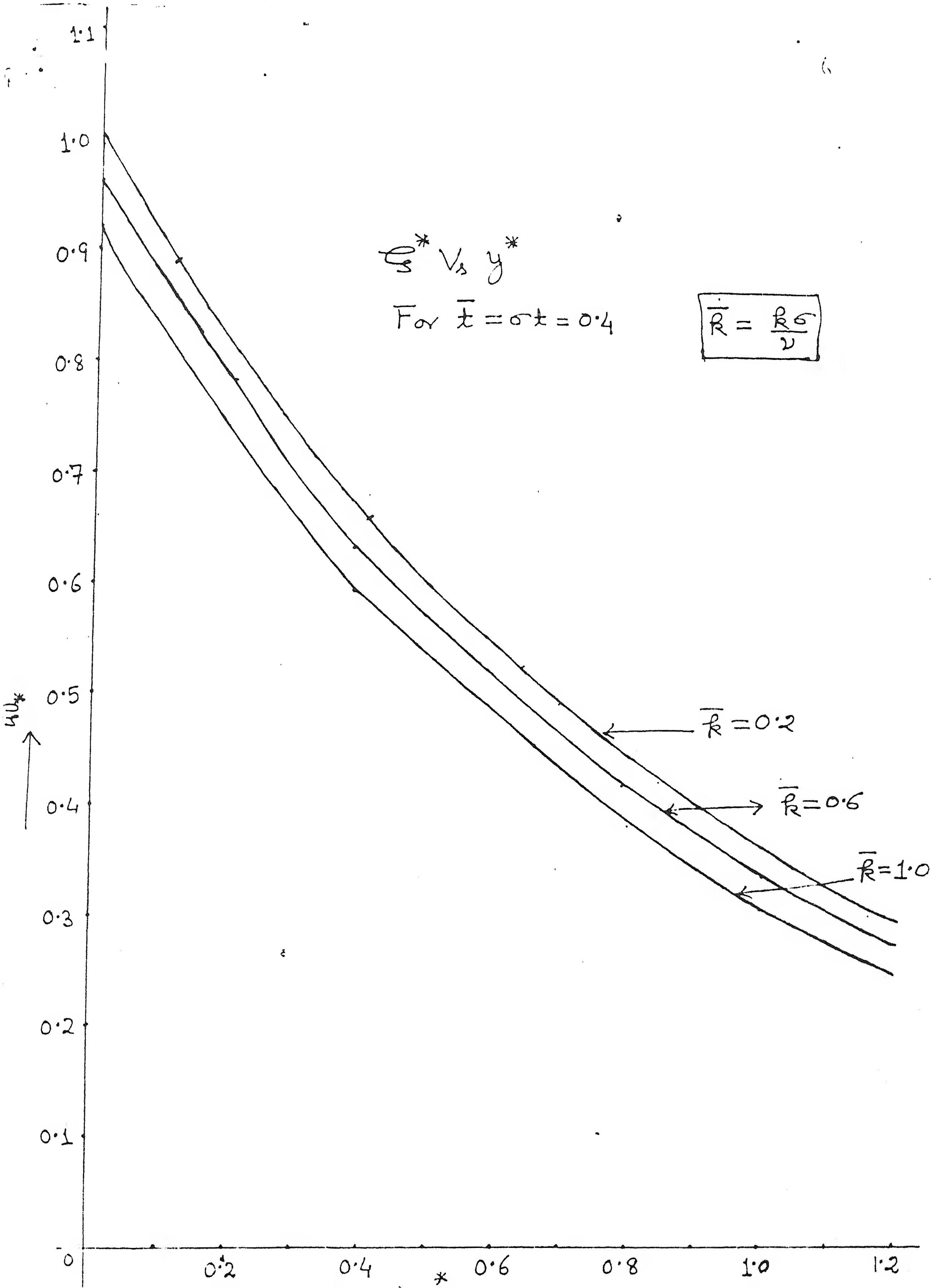
$\bar{t} = 0.8$

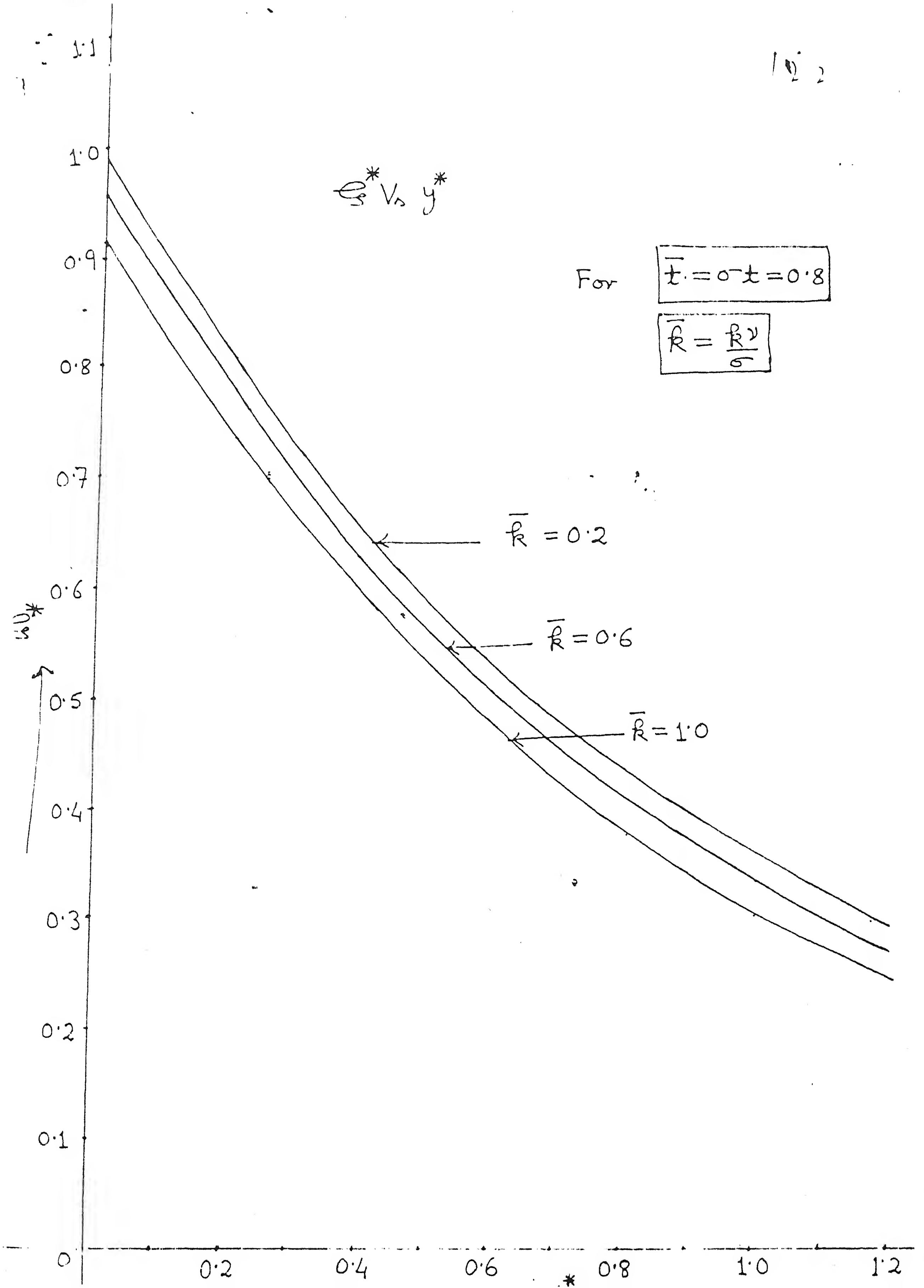
$\bar{k}$	$\xi^* \backslash \eta^*$	0.0	0.2	0.4	0.6	0.8	1.0	1.2
0.2	ξ^*	0.9938	0.8127	0.6648	0.5437	0.4447	0.3638	0.2975
0.6	ξ^*	0.9599	0.7797	0.6334	0.5145	0.4179	0.3395	0.2758
1.0	ξ^*	0.9184	0.7377	0.5926	0.4849	0.3824	0.3071	0.2467

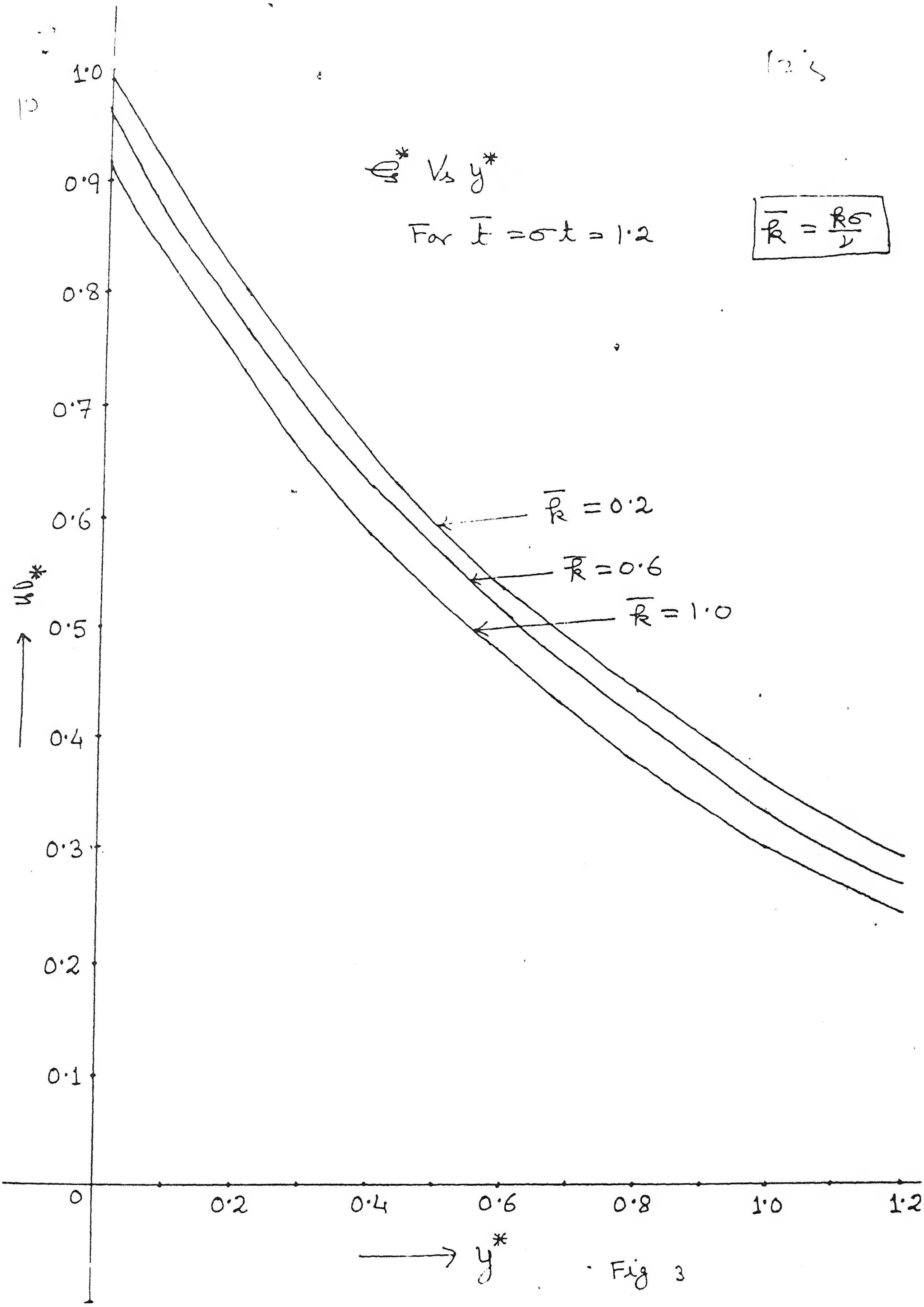
TABLE 3

$\bar{t} = 1.2$

k	$\xi^* \backslash \eta^*$	0.0	0.2	0.4	0.6	0.8	1.0	1.2
0.2	ξ^*	0.9928	0.8121	0.6643	0.5433	0.4444	0.3635	0.2973
0.6	ξ^*	0.9579	0.7781	0.6321	0.5135	0.4171	0.3388	0.2752
1.0	ξ^*	0.9156	0.7355	0.5908	0.4834	0.3812	0.3062	0.2460


C_s^* Vs y^*
For $\bar{t} = \sigma t = 0.4$
$\bar{k} = \frac{k\sigma}{\nu}$
$\bar{k} = 0.2$
$\bar{k} = 0.6$
$\bar{k} = 1.0$
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.2
0.4
0.6
0.8
1.0
1.2


$\theta_s^* \text{ Vs } y^*$
For $\bar{t} = \sigma t = 0.8$
$\bar{k} = \frac{k\nu}{\sigma}$
$\bar{k} = 0.2$
$\bar{k} = 0.6$
$\bar{k} = 1.0$
θ_s^*
y^*
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
0.2
0.4
0.6
0.8
1.0
1.2



θ^* Vs y^*
For $\bar{t} = \sigma t = 1.2$
$\bar{k} = \frac{k\sigma}{\nu}$
$\bar{k} = 0.2$
$\bar{k} = 0.6$
$\bar{k} = 1.0$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.2
0.4
0.6
0.8
1.0
1.2
θ^*
y^*

Fig 3

CONCLUSION

The effect of permeability on the vorticity of flow of a viscous fluid in a porous medium due to an oscillating flat plate has been studied through the calculations given in tables 1-3 and figure 1-3.

It is clear from above observations that the vorticity is maximum at the plate which is self-explanatory because of oscillations in it. As we move away the plate the value of vorticity lessens but we may practically not expect any region of irrotationality in its vicinity. With increase in time although the vorticity lessens but very slightly. Also the increase in permeability of the porous medium does not have any marked effect on it. Although, with the increase in permeability the net value of vorticity decreases slightly, but its mode of decrease with y* remains almost same. So why, the vorticity profiles are almost parallel.

5.5 PAPER - II

ON THE VORTICITY OF HYDROMAGNETIC VISCO-ELASTIC FLUID NEAR OSCILLATING POROUS FLAT PLATE

ABSTRACT

In this paper an attempt has been made to find the vorticity of Hydromagnetic visco-elastic fluid

(Kuvshinki's fluid) with small electrical conductivity near an infinite insulated porous flat plate oscillating hormonically in its own plane in the presence of a transverse magnetic field of uniform strength fixed relative to the fluid. Laplace transform technique has been used for the solution of the problem. Small uniform suction has been imposed along the surface of the plate. Numerical calculations has been carried out for the vorticity field for different values of suction parameter, magnetic parameter and elastic parameter at T=0 and T= π/2 and vorticity profiles obtained and result discussed critically.

INTRODUCTION

Stokes [12] and Rayleigh [8] investigated the flow about an infinite flat wall executing linear harmonic oscillations parallel to itself. Rossow [9] discussed the impulsive motion of the infinite plate in a viscous incompressible conducting fluid in the presence of an external magnetic field. Ong and Nicholls [6] extended the method to obtain the flow of an electrically conducting fluid near an infinite flat wall which oscillates in its own plane. Chaudhary [2] has given a detailed study of the motion of a viscous incompressible and electrically conducting fluid near an infinite oscillating flat plate in the presence of an uniform

magnetic field relative to the fluid.

In this paper we have made an effort to find the vorticity of the hydromagnetic flow of a visco-elastic fluid near an oscillating porous flat plate. The effect of various parameters on the vorticity of hydromagnetic flow has been discussed numerically and graphically.

FORMULATION AND SOLUTION OF THE PROBLEM

Let us consider an infinite insulated porous flat plate executing linear harmonic oscillations in its own plane in presence of an externally applied transverse magnetic field of uniform strength H_o fixed relative to the fluid. As the plate is infinite in length and uniform suction is imposed over it, the physical variables depend only on y, the coordinate perpendicular to the wall and t, the time. The pressure p in the fluid assumed constant.

The equations governing the flow are

$$\rho\left(\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y}\right) = \frac{\partial p_{xy}}{\partial y} - \sigma \mu_e^2 H_o^2 u \qquad \text{.........(5.12)}$$

$$\frac{\partial v}{\partial y} = 0 \qquad \text{(Continuity equation) ......(5.13)}$$

From (5.12) and (5.13) we have

$$p_{xy} + \lambda\left(\frac{\partial p_{xy}}{\partial t} + v\frac{\partial p_{xy}}{\partial y}\right) = \mu\frac{\partial u}{\partial y} \qquad \text{.........(5.14)}$$

Here λ is the relaxation time, signifies that the shear stress decays are exp $(-t/\lambda)$ when the motion stops suddenly.

If V_0 represents the velocity of suction or injection at the plate, the equation (5.13) with the condition $y=0$, $V=V_0$ leads to the result $V=V_0$ every where.

Now eleminating p_{xy} between (5.12) and (5.13), We get

$$\left[1+\lambda\left(\frac{\partial}{\partial t}+V_0\frac{\partial}{\partial y}\right)\right]\left[\frac{\partial u}{\partial t}+V_0\frac{\partial u}{\partial y}+mu\right]=\nu\frac{\partial^2 u}{\partial y^2} \qquad \text{.............(5.15)}$$

Where u is the velocity parallel to the plate, ρ the density, $\nu=\frac{\mu}{\rho}$ the kinematic viscosity, σ the electrical conductivity and μ_e the magnetic permeability.

The term $\left(mu=\frac{\sigma}{\rho}\mu_e^2 H_0^2 u\right)$ represents the Ponder motive force parallel to the plate. Here the magnetic Reynolds number R_σ is considered small so that the induced magnetic field due to the flow may be neglected with respect to the applied magnetic field. The boundary conditions of the problem are

$$\left.\begin{array}{l} y=0\ ;\ u=u_0\cos nt \\ y\to\infty\ ;\ u\to 0 \end{array}\right\} \qquad \text{........(5.16)}$$

Let us assume that

$$\left.\begin{array}{l} T=nt,\ y=n\sqrt{\frac{\nu}{n}},\ M=\frac{m}{n},\ n\lambda=\psi \\ \bar{V}=\frac{V_0}{\sqrt{n\nu}},\ \begin{array}{l} V_0>0 \text{ injection} \\ V_0<0 \text{ suction} \end{array} \end{array}\right\} \qquad \text{.....(5.17)}$$

Accordingly the equation (5.15) reduces to

$$\left[1+\Psi\left(\frac{\partial}{\partial T}+\bar{V}_0\frac{\partial}{\partial \eta}\right)\right]\left[\frac{\partial u}{\partial T}+\bar{V}_0\frac{\partial u}{\partial \eta}+Mu\right]=\frac{\partial^2 u}{\partial \eta^2} \qquad \text{.......(5.18)}$$

The transformed boundary conditions are :

$$\left.\begin{array}{lll} \eta = 0 & : & u = u_0 \cos T \\ \eta \to \infty & : & u \to 0 \end{array}\right\} \qquad \text{........(5.19)}$$

Initially, it is assumed that, the fluid is at rest,

i.e.,

$$u = 0 \;, \quad \frac{\partial u}{\partial T} = 0 \quad \text{at} \quad T = 0$$

Applying Laplace Transform on each term of equation (5.18) we get

$$\frac{d^2\tilde{u}}{d\eta^2} - \frac{\bar{V}_0(1+2p\Psi+\Psi M)}{(1-\Psi V_0^2)}\frac{d\tilde{u}}{d\eta} - \frac{(p+M+p^2\Psi^2+\Psi M p)\tilde{u}}{(1-\Psi V_0^2)} = 0$$

$$\text{........(5.20)}$$

Where $\displaystyle \tilde{u} = \int_0^\infty u \,.\, \exp(-pT)\, dT$

The solution of equation (5.20) is given by

$$\tilde{u} = \left(\frac{p}{1+p^2}\right)\exp\{-\eta(p+\delta)b\}\exp\left[-a\eta\sqrt{\{(p+\beta)(p+\gamma)\}}\right] \text{........(5.21)}$$

where

$$\left.\begin{array}{l} a = \dfrac{\sqrt{\Psi}}{(1-\Psi\bar{V}_0^2)}\,, \quad b = \dfrac{\Psi\bar{V}_0}{(1-\Psi\bar{V}_0^2)}\,, \quad \delta = \dfrac{M\Psi+1}{2\Psi} \\ \beta = \dfrac{1}{2\Psi}\left[(M\Psi+1)-(M\Psi-1)\sqrt{(1-\Psi\bar{V}_0^2)}\right] \\ \gamma = \dfrac{1}{2\Psi}\left[(M\Psi+1)+(M\Psi-1)\sqrt{(1-\Psi\bar{V}_0^2)}\right] \\ \Psi\bar{V}_0^2 < 1\,, \quad M\Psi > 1 \end{array}\right\} \text{........(5.22)}$$

Using Laplace inversion, theorem, we get

$$u(n,T) = \frac{u_0}{2\pi i}\int_{\nu-i\infty}^{\nu+i\infty} \frac{p}{1+p^2}\, e^{pT} . \exp\{-nb(p+\delta)\}.$$

$$\exp[-an\sqrt{\{(p+\beta)(p+\nu)\}}]\,dp \quad \ldots\ldots(5.23)$$

Using the theory of residues, the velocity distribution $u(n,T)$ is given by [4] as.

$$u(n,T) = u_0 \exp\{-n(b\delta + a\rho\cos\theta)\}\cos\{T - n(b + a\rho\sin\theta)\} \quad ..(5.24)$$

Where

$$\rho\cos\theta = \frac{1}{\sqrt{2}}\left[\sqrt{(\beta^2+\nu^2+\beta^2\nu^2+1)} + \beta\nu - 1\right]^{1/2}$$

$$\rho\sin\theta = \frac{1}{\sqrt{2}}\left[\sqrt{(\beta^2+\nu^2+\beta^2\nu^2+1)} + 1 - \beta\nu\right]$$

The vorticity of flow (5.23) is given by

$$\zeta = u_0 \exp\{-n(b\delta + a\rho\cos\theta)\}\left[(b + a\rho\sin\theta)\sin\{T - n(b + a\rho\sin\theta)\}\right.$$

$$\left. -(b\delta + a\rho\cos\theta)\cos\{T - n(b + a\rho\sin\theta)\}\right] \quad \ldots\ldots\ldots(5.25)$$

CASE-I When $\psi = 0$, $V_0 = 0$, $M = 0$

We have $\beta \to \infty$, $\nu \to 0$

The velocity field (5.24) reduces to

$$u(n,T) = u_0 \exp(-n/\sqrt{2})\cos(T - n/\sqrt{2}) \quad \ldots\ldots\ldots(5.26)$$

and the vorticity of flow (5.26) in this case becomes

$$\zeta = \frac{U_0}{\sqrt{2}} \exp(-n/\sqrt{2})\left[\sin\left(T-\frac{n}{\sqrt{2}}\right) - \cos(T-n/\sqrt{2})\right]$$

or

$$\zeta^* = \frac{\sqrt{2}\,\zeta}{u_0} = \exp\left(-\frac{n}{\sqrt{2}}\right)\left[\sin\left(T-\frac{n}{\sqrt{2}}\right)-\cos\left(T-\frac{n}{\sqrt{2}}\right)\right] \quad \ldots\ldots(5.27)$$

<u>CASE-II</u> When $\Psi = 0$, $\bar{V} = 0$, $M \neq 0$, Then

$$\beta \to \infty, \quad \nu \to M$$

and the velocity field in this case is

$$u(n,T) = u_0 \exp\left[-\frac{n}{\sqrt{2}}\left\{\sqrt{(1+M^2)} + M\right\}^{1/2}\right]$$

$$\cos\left[T - \frac{n}{2}\left\{\sqrt{(1+M^2)} - M\right\}^{1/2}\right] \quad \ldots\ldots(5.28)$$

The vorticity of flow in this case is given by

$$\zeta^* = \frac{\sqrt{2}\,\zeta}{u_0} = \exp\left[-\frac{n}{2}\left\{\sqrt{(1+M^2)} + M_1\right\}^{1/2}\right] \times$$

$$\left[\left(\sqrt{(1+M^2)} - M\right)^{1/2} \sin\left\{T - \frac{n}{2}\left(\sqrt{(1+M^2)} - M\right)^{1/2}\right\}\right.$$

$$\left. - \left(\sqrt{(1+M^2)} + M\right)^{1/2} \cos\left\{T - \frac{n}{2}\left(\sqrt{(1+M^2)} - M\right)^{1/2}\right\}\right] \quad \ldots\ldots(5.29)$$

<u>CASE-III</u> When $M_1 \neq 0$, $\bar{V}_0 \neq 0$, $\Psi = 0$, we have

$$\beta \to \infty, \quad \nu \to M + \frac{\bar{V}_0^2}{4}$$

and

$$\rho\cos\theta = \frac{1}{2\sqrt{2}}\left[\sqrt{\left\{16 + (\bar{V}_0^2 + 4M)^2\right\}} + (\bar{V}_0^2 + 4M)\right]^{1/2}$$

$$\rho\sin\theta = \frac{1}{2\sqrt{2}}\left[\sqrt{\left\{16 + (\bar{V}_0^2 + 4M)^2\right\}} - (\bar{V}_0^2 + 4M)\right]^{1/2}$$

and the velocity field in this case is given by

$$u(\eta,T) = u_0 \exp\left[\frac{\bar{V}_0}{2}\eta - \frac{\eta}{2\sqrt{2}}\left\{\sqrt{\{16+(\bar{V}_0^2+4M)^2\}}+(\bar{V}_0^2+4M)\right\}^{1/2}\right]\times$$

$$\cos\left[T-\frac{\eta}{2\sqrt{2}}\left\{\sqrt{\{16+(\bar{V}_0^2+4M)^2\}}-(\bar{V}_0^2+4M)\right\}^{1/2}\right] \quad (5.30)$$

The vorticity of flow in this case is given by

$$\xi^* = \frac{\sqrt{2}\,\xi}{u_0} = \exp\left[\frac{\bar{V}_0\eta}{2} - \frac{\eta}{2\sqrt{2}}\left\{\sqrt{\{16+(\bar{V}_0^2+4M)^2\}}+(\bar{V}_0^2+4M)\right\}^{1/2}\right]$$

$$\times\left[\left(\frac{\bar{V}_0}{\sqrt{2}} - \frac{1}{2}\left\{\sqrt{\{16+(\bar{V}_0^2+4M)^2\}}+(\bar{V}_0^2+4M)\right\}^{1/2}\times\right.\right.$$

$$\cos\left(T-\frac{\eta}{2\sqrt{2}}\left\{\sqrt{\{16+(\bar{V}_0^2+4M)^2\}}-(\bar{V}_0^2+4M)\right\}^{1/2}\right)$$

$$+\frac{1}{2}\left\{\sqrt{\{16+(\bar{V}_0^2+4M)^2\}}-(\bar{V}_0^2+4M)\right\}^{1/2}\times$$

$$\sin\left(T-\frac{\eta}{2\sqrt{2}}\left\{\sqrt{16+(\bar{V}_0^2+4M)^2}\right\}\right.$$

$$\left.\left.-(\bar{V}_0^2+4M)\right\}^{1/2}\right)\Bigg] \quad \ldots\ldots\ldots\ldots (5.31)$$

TABLE 1

$\Psi = 0$, $M = 0$, $T = 0$

V_0	$\xi^* \backslash \eta$	0.0	0.5	1.0	1.5	2.0	2.5	3.0
0	ξ^*	-1.0000	-0.9018	-0.6952	-0.4712	-0.2431	-0.2008	-0.1648
2	ξ^*	-0.1408	-0.1720	-0.1517	-0.1161	-0.0814	-0.0535	-0.0333

TABLE 2

$\psi = 0$, $M = 0$, $T = \pi/2$

$\bar{V}_0$	ξ_y \ η	0.0	0.5	1.0	1.5	2.0	2.5	3.0
0	ξ_y	0.000	0.2431	0.3203	0.3021	0.2401	0.1674	0.10
2	ξ_y	0.645	0.2445	0.1911	0.0923	0.0387	0.0117	-0.00

Table 3

$\psi = 0$, $M = 2$, $T = 0$

$\bar{V}_0$	ξ_y \ η	0.0	0.5	1.0	1.5	2.0	2.5	3.0
0	ξ_y	-2.0582	-1.0196	-0.4903	-0.2288	-0.1034	-0.0450	-0.0187
2	ξ_y	-1.0682	-0.7639	-0.5348	-0.3668	-0.2465	-0.1709	-0.1043

TABLE 4

$\psi = 0$, $M = 2$, $T = \pi/2$

$\bar{V}_0$	ξ_y \ η	0.0	0.5	1.0	1.5	2.0	2.5	3.0
0	ξ_y^*	0.4859	0.0613	-0.0550	-0.0726	-0.0506	-0.0270	-0.0192
2	ξ_y^*	0.4028	0.1694	0.0406	-0.0245	-0.0523	-0.0595	-0.0561

TABLE 5

$\psi = 2$, $M = 0$, $T = 0$

$\bar{V}_0$	ξ_y \ η	0.0	0.5	1.0	1.5	2.0	2.5	3.0
0	ξ_y^*	-0.9717	2.0265	1.7087	-0.2418	-1.4964	-1.2095	-0.0129
2	ξ_y^*	-0.5068	-2.3169	-2.6871	-1.7011	-0.0541	1.1629	1.5508

TABLE 6

$\psi = 2$, $M_1 = 0$, $T = \pi/2$

$\bar{V}_0$	ξ_y \ η	0.0	0.5	1.0	1.5	2.0	2.5	3.0
0	ξ_y^*	-4.1163	-1.8479	0.9067	1.6709	0.5708	-0.7830	-0.8301
2	ξ_y^*	-3.0911	-1.7869	0.2066	1.7488	1.8751	1.2565	0.2951

TABLE 7

$\psi = 2$, $M_1 = 2$, $T = 0$

$\bar{V}_0$	$\zeta_y \backslash \eta$	0.0	0.5	1.0	1.5	2.0	2.5	3.0
0	ζ_y^*	-4.4721	0.6751	0.6711	0.1857	0.0617	-0.0235	-0.0027
2	ζ_y^*	-3.0936	0.7777	0.0900	0.0173	-0.0541	-0.0540	-0.0315

TABLE 8

$\psi = 2$, $M_1 = 2$, $T = \pi/2$

$\bar{V}_0$	$\zeta_y \backslash \eta$	0.0	0.5	1.0	1.5	2.0	2.5	3.0
0	ζ_y^*	-4.4721	-0.2478	0.5779	0.0779	0.0217	0.0195	-0.0005
2	ζ_y^*	-3.0673	-1.8464	-0.2129	-0.0214	0.0141	-0.0139	-0.0265

The effect of suction, magnetic field parameter and elastic parameter on the vorticity of the flow, at $T=0$ and $T=\pi/2$ has been shown in figures. It is clear that variation of vorticity is not uniform in the flow and it markebly depend on ν_0, ψ, M and T. In almost all the cases the vorticity tends to become zero. In some cases the region of irrotationality lies near the plate (the distance depends on the parameters) and in others it lessens as we move away from the plate. Obviously the elastic parameter and the suction or injection velocity govern the vorticity of flow while the introduction of magnetic field adds more complications to it.

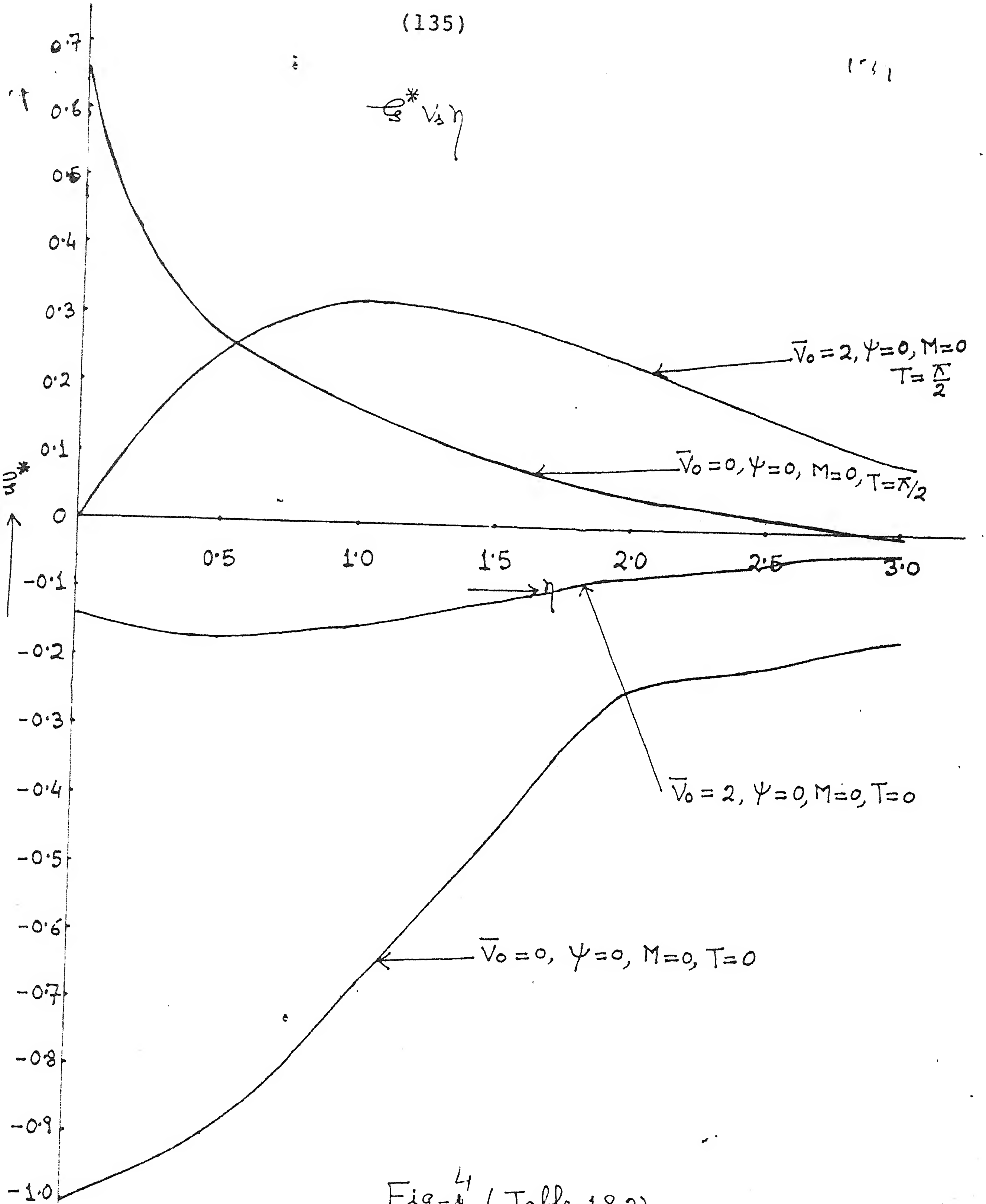


Fig-4 (Table 1 & 2)

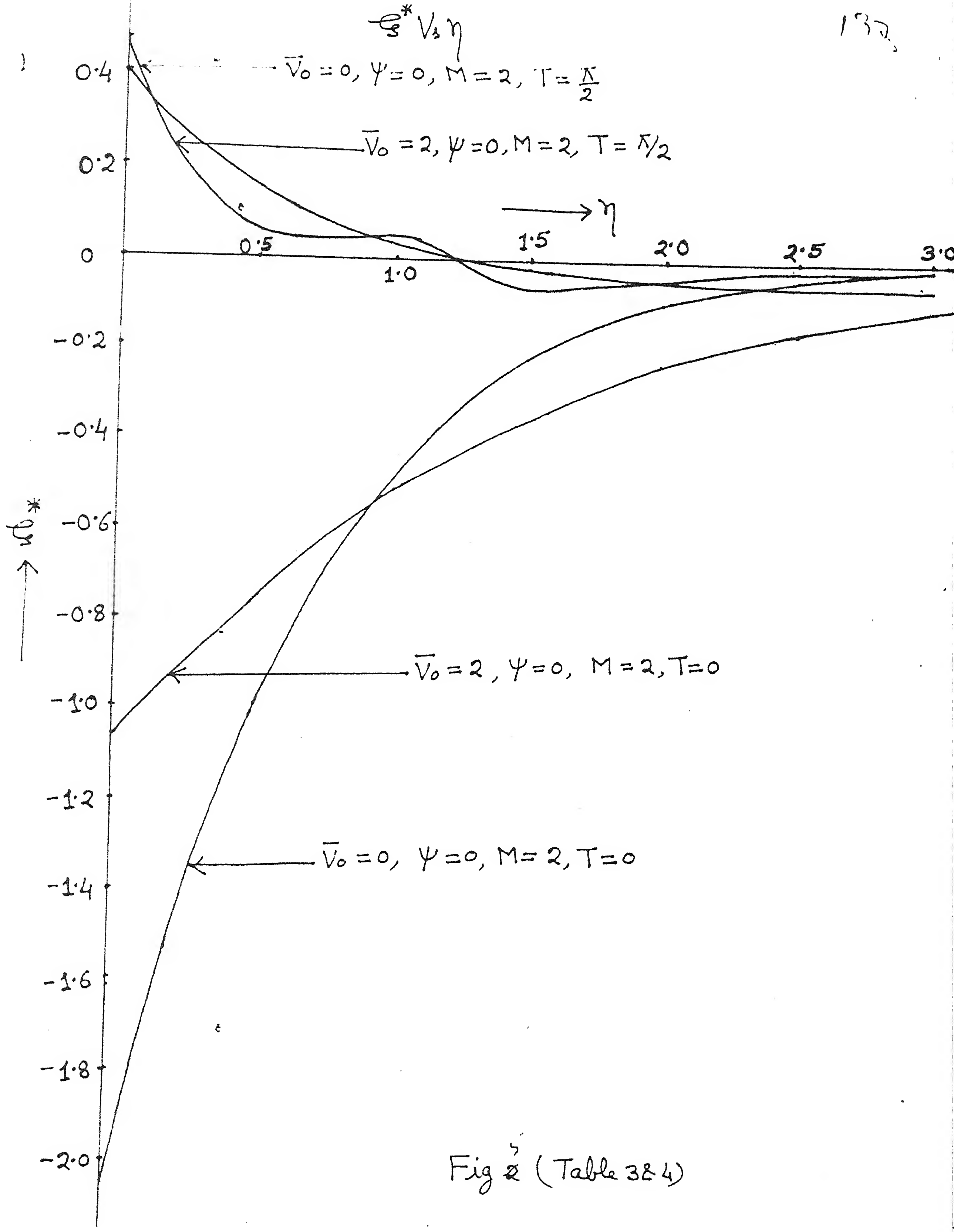


Fig 5 (Table 3 & 4)

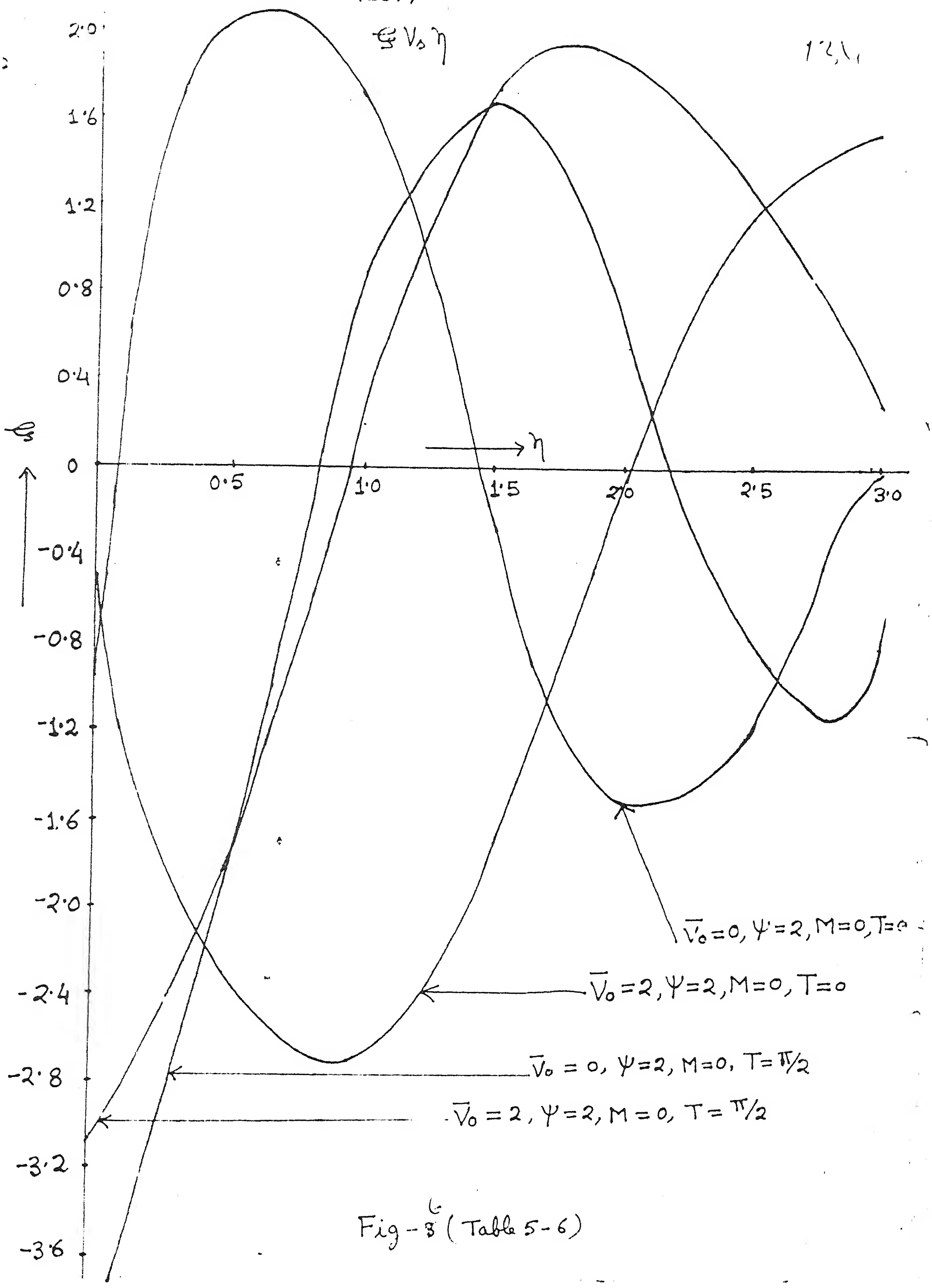


Fig-6 (Table 5-6)

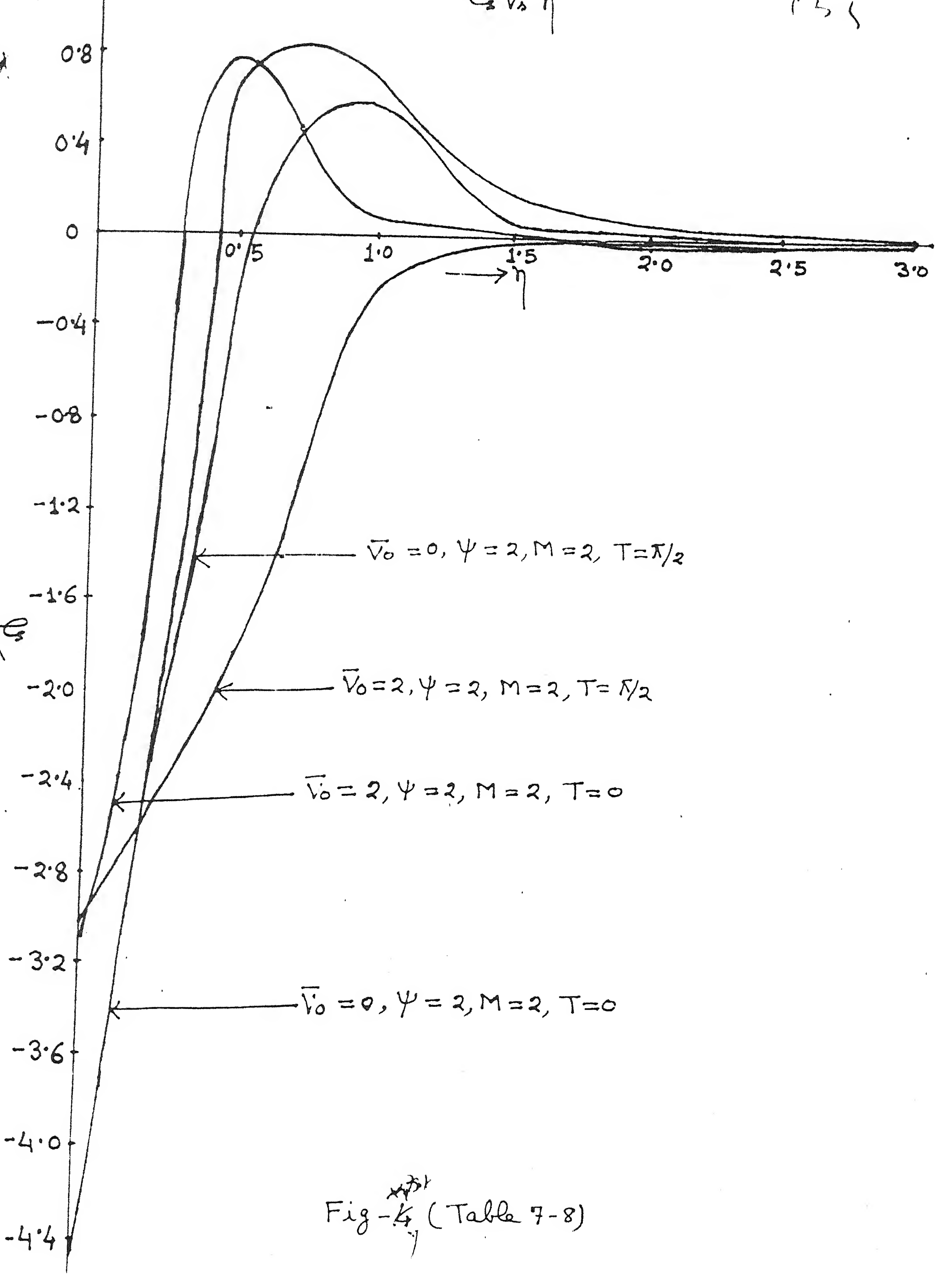


Fig-4 (Table 7-8)

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